Question 4 (14 marks)

Patricia is a gardener and she owns a garden nursery. She grows and sells basil plants and coriander plants.

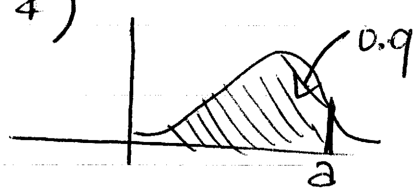
The heights, in centimetres, of the basil plants that Patricia is selling are distributed normally with a mean of 14 cm and a standard deviation of 4 cm. There are 2000 basil plants in the nursery.

a. Patricia classifies the tallest 10 per cent of her basil plants as **super**.

What is the minimum height of a super basil plant, correct to the nearest millimetre? 1 mark

CAS
invNorm(0.9, 14, 4)

$H \overset{d}{=} N(\mu = 14, \sigma = 4)$



$a \simeq 19.12$ cm

$\simeq 191$ mm

Patricia decides that some of her basil plants are not growing quickly enough, so she plans to move them to a special greenhouse. She will move the basil plants that are less than 9 cm in height.

b. How many basil plants will Patricia move to the greenhouse, correct to the nearest whole number? 2 marks

$\Pr(H \leq 9) \simeq 0.10565$

CAS
normCdf(−∞, 9, 14, 4)

0.10565×2000

$\simeq 211$ plants

The heights of the coriander plants, x centimetres, follow the probability density function $h(x)$, where

$$h(x)=\begin{cases}\frac{\pi}{100}\sin\left(\frac{\pi x}{50}\right) & 0<x<50\\ 0 & \text{otherwise}\end{cases}$$

c. State the mean height of the coriander plants. 1 mark

$$E(x) = \int_0^{50} \frac{x.\pi}{100}\sin\left(\frac{\pi x}{50}\right)dx$$

$$= 25 \text{ cm}$$

Patricia thinks that the smallest 15 per cent of her coriander plants should be given a new type of plant food.

d. Find the maximum height, correct to the nearest millimetre, of a coriander plant if it is to be given the new type of plant food. 2 marks

$$\int_0^{p} \frac{\pi}{100}\sin\left(\frac{\pi x}{50}\right)dx = 0.15 \quad , 0<p<50.$$

Solving : $p = 12.6592$ cm

$\simeq 12.7$ cm

Patricia also grows and sells tomato plants that she classifies as either **tall** or **regular**. She finds that 20 per cent of her tomato plants are tall.

A customer, Jack, selects n tomato plants at random.

e. Let q be the probability that at least one of Jack's n tomato plants is tall.

Find the minimum value of n so that q is greater than 0.95. 2 marks

Let X = no. of tall plants in n plants

$X = 0, 1, 2, \ldots n$

$X \stackrel{d}{=} Bi(n, 0.2)$

$Pr(X \geq 0) = 0.95$

$1 - Pr(X=0) = 0.95$

$\therefore Pr(X=0) = 0.05$

$\therefore \binom{n}{0}(0.2)^0(0.8)^n = 0.05$

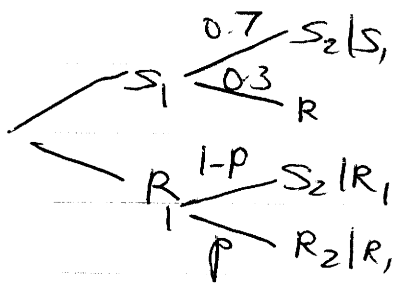
$n = 13.42$

$\therefore$ Min value of n is 14.

In another section of the nursery, a craftsman makes plant pots. The pots are classified as **smooth** or **rough**.

The craftsman finishes each pot before starting on the next. Over a period of time, it is found that if one plant pot is smooth, the probability that the next one is smooth is 0.7, while if one plant pot is rough, the probability that the next one is rough is p, where $0 < p < 1$. The value of p stays fixed for a week at a time, but can vary from week to week. The first pot made each week is always a smooth pot.

f. **i.** Find, in terms of p, the probability that the **third** pot made in a given week is smooth. 2 marks



$$\begin{bmatrix}0.7 & 1-p\\ 0.3 & p\end{bmatrix}^2\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}Pr(S_3)\\ Pr(R_3)\end{bmatrix}$$

$$=\begin{bmatrix}\frac{79}{100}-\frac{3p}{10}\\ \frac{3p}{10}+\frac{21}{100}\end{bmatrix} \quad \therefore Pr(S_3) = 0.79 - 0.3p$$

ii. In one particular week, the probability that the **third** pot made is smooth is 0.61.

Calculate the value of p in this week. 2 marks

$$0.79 - 0.3p = 0.61$$
$$\therefore 0.18 = 0.3p$$
$$p = \frac{18}{30}$$
$$p = 0.6$$

g. If, in another week, $p = 0.8$, find the probability that the **fifth** pot made that week is smooth. 2 marks

$$\begin{bmatrix}0.7 & 0.2\\ 0.3 & 0.8\end{bmatrix}^4\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}\frac{7}{16}\\ \frac{9}{16}\end{bmatrix}$$

$$\therefore Pr(S_5) = \frac{7}{16}.$$

Question 3

Steve, Katerina and Jess are three students who have agreed to take part in a psychology experiment. Each student is to answer several sets of multiple-choice questions. Each set has the same number of questions, n, where n is a number greater than 20. For each question there are four possible options (A, B, C or D), of which only one is correct.

a. Steve decides to guess the answer to every question, so that for each question he chooses A, B, C or D at random.

Let the random variable X be the number of questions that Steve answers correctly in a particular set.

i. What is the probability that Steve will answer the first three questions of this set correctly?

$\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

ii. Find, to four decimal places, the probability that Steve will answer at least 10 of the first 20 questions of this set correctly.

$C \stackrel{d}{=} Bi\,(n = 20, p = \frac{1}{4})$ C = no. of correct answers in 20 qns

$Pr(C \geq 10) = 0.0139$

iii. Use the fact that the variance of X is $\frac{75}{16}$ to show that the value of n is 25.

$Var(X) = np(1-p)$ $n\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{75}{16}$

$\therefore \frac{3n}{16} = \frac{75}{16}$

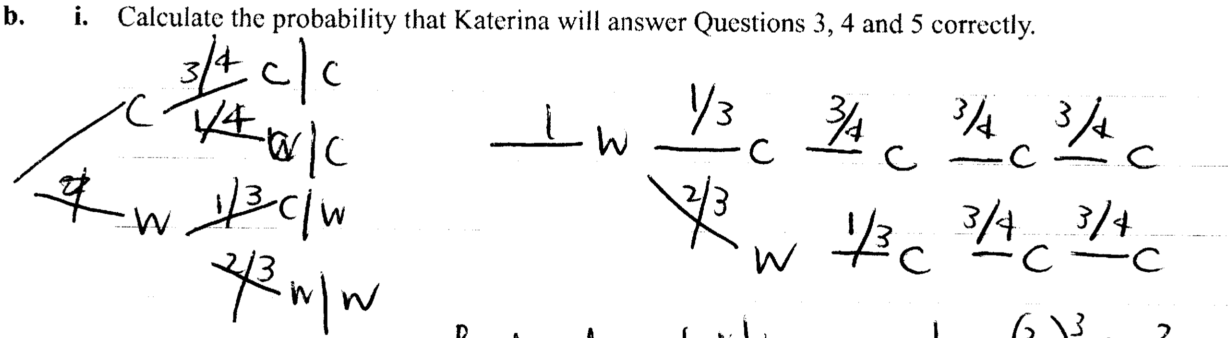
$\therefore n = 25$

1 + 2 + 1 = 4 marks

If Katerina answers a question correctly, the probability that she will answer the next question correctly is $\frac{3}{4}$. If she answers a question incorrectly, the probability that she will answer the next question incorrectly is $\frac{2}{3}$.

In a particular set, Katerina answers Question 1 incorrectly.

b. **i.** Calculate the probability that Katerina will answer Questions 3, 4 and 5 correctly.



$$\text{Required probability} = \frac{1}{3} \times \left(\frac{3}{4}\right)^3 + \frac{2}{3} \times \frac{1}{3} \times \left(\frac{3}{4}\right)^2$$

$$= \frac{17}{64}$$

ii. Find the probability that Katerina will answer Question 25 correctly. Give your answer correct to four decimal places.

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{3} \\ \frac{1}{4} & \frac{2}{3} \end{bmatrix}^{24} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5714 \\ 0.4286 \end{bmatrix}$$

$\therefore$ Required probability $= 0.5714$.

3 + 2 = 5 marks

c. The probability that Jess will answer any question correctly, independently of her answer to any other question, is p ($p > 0$). Let the random variable Y be the number of questions that Jess answers correctly in any set of 25.

If $\Pr(Y > 23) = 6\Pr(Y = 25)$, show that the value of p is $\frac{5}{6}$.

$$\Pr(Y=24) + \Pr(Y=25) = 6\Pr(Y=25)$$

$$\therefore \binom{25}{24} p^{24}(1-p) + p^{25} = 6p^{25}$$

$$\therefore 25p^{24}(1-p) = 5p^{25}$$

$$\therefore 1-p = \frac{1}{5}p$$

$$\therefore 1 = \frac{6p}{5}$$

$$p = \frac{5}{6}$$

2 marks

d. From these sets of 25 questions being completed by many students, it has been found that the time, in minutes, that any student takes to answer each set of 25 questions is another random variable, W, which is **normally distributed** with mean a and standard deviation b.

It turns out that, for Jess, $\Pr(Y \geq 18) = \Pr(W \geq 20)$ and also $\Pr(Y \geq 22) = \Pr(W \geq 25)$.

Calculate the values of a and b, correct to three decimal places.

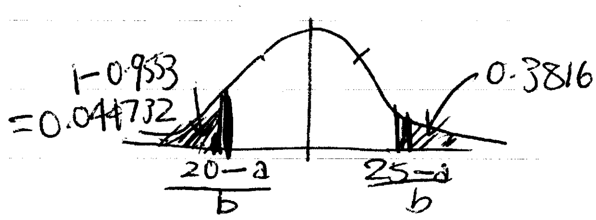
$$W \stackrel{d}{=} N(\mu = a, \sigma = b)$$

$$\Pr(Y \geq 18) = 0.955268\ldots \longleftarrow \text{CAS } \mathrm{binomcdf}(25, \tfrac{5}{6}, 18, 25)$$

$$\Pr(Y \geq 22) = 0.381566\ldots \longleftarrow \text{CAS } \mathrm{binomcdf}(25, \tfrac{5}{6}, 22, 25)$$

$$\Pr(W \geq 20) = 0.955268$$

$$\Pr(W \geq 25) = 0.381566$$



$$\frac{20-a}{b} = -1.69823\ldots$$

$$\frac{25-a}{b} = 0.30137\ldots$$

$$a = 24.246,\ b = 2.500$$

4 marks

Question 2

In a chocolate factory the material for making each chocolate is sent to one of two machines, machine A or machine B.

The time, X seconds, taken to produce a chocolate by machine A, is normally distributed with mean 3 and standard deviation 0.8.

The time, Y seconds, taken to produce a chocolate by machine B, has the following probability density function.

$$f(y)=\begin{cases}0 & y<0\\ \dfrac{y}{16} & 0\le y\le 4\\ 0.25e^{-0.5(y-4)} & y>4\end{cases}$$

a. Find correct to four decimal places

i. $\Pr(3 \le X \le 5)$

$$X \overset{d}{=} N(\mu=3, \sigma=0.8)$$

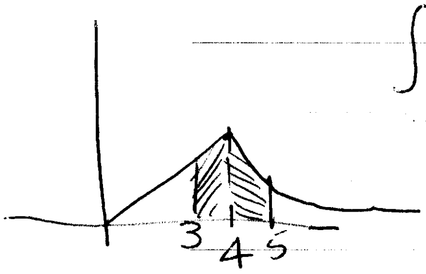
$$\Pr(3 \le X \le 5) = 0.4938$$

CAS

normalcdf(3, 5, 3, 0.8)

ii. $\Pr(3 \le Y \le 5)$

$$\int_3^4 \frac{y}{16}\,dy + \int_4^5 0.25e^{-0.5(y-4)}\,dy \approx 0.4155$$



1 + 3 = 4 marks

b. Find the mean of Y, correct to three decimal places.

$$E(Y) = \int_0^4 \frac{y^2}{16}\,dy + \int_4^\infty 0.25y\,e^{-0.5(y-4)}\,dy$$

$$= 4.333$$

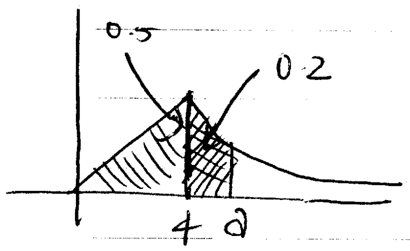
3 marks

c. **i.** Find the median of Y.

$$\int_0^4 \frac{y}{16}\,dy = \left[\frac{y^2}{32}\right]_0^4 = 0.5$$

$\therefore$ Median $= 4$

ii. Find the value of a, correct to two decimal places, such that $\Pr(Y \le a) = 0.7$.



$$\int_4^a 0.25\, e^{-0.5(y-4)}\,dy = 0.2$$

Solving: $a = 5.02$

1 + 2 = 3 marks

d. It can be shown that $\Pr(Y \le 3) = \frac{9}{32}$. A random sample of 10 chocolates **produced by machine *B*** is chosen. Find the probability, correct to four decimal places, that exactly 4 of these 10 chocolates took 3 or less seconds to produce.

Let X = no. of chocolates that took less than 3 seconds to produce

$X \stackrel{d}{=} Bi\left(n = 10, p = \frac{9}{32}\right)$

$\Pr(X = 4) = \binom{10}{4}\left(\frac{9}{32}\right)^4\left(\frac{23}{32}\right)^6$ [OR CAS binompdf$\left(10, \frac{9}{32}, 4\right)$]

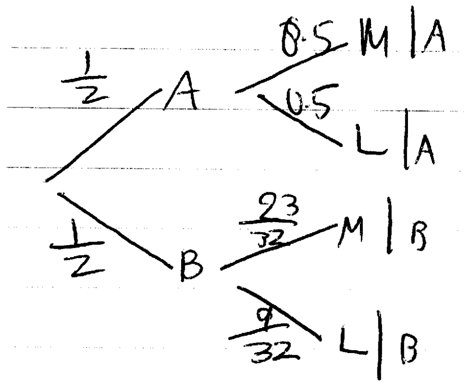
$= 0.1812$

2 marks

All of the chocolates produced by machine A and machine B are stored in a large bin. There is an equal number of chocolates from each machine in the bin.

It is found that if a chocolate, produced by either machine, takes longer than 3 seconds to produce then it can easily be identified by its darker colour.

e. A chocolate is selected at random from the bin. It is found to have taken longer than 3 seconds to produce. Find, correct to four decimal places, the probability that it was produced by machine A.



M = more than 3 sec
L = less than 3 sec

$$\Pr(A|M) = \frac{\Pr(A \cap M)}{\Pr(M)}$$

$$= \frac{\frac{1}{2} \times 0.5}{\frac{1}{2} \times 0.5 + \frac{1}{2} \times \frac{23}{32}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{23}{32}}$$

$$= \frac{\frac{1}{2}}{\frac{39}{32}}$$

$$= \frac{16}{39}$$

$$\simeq 0.4103.$$

For A, $\Pr(X > 3) = 0.5$ since $\mu = 3$

3 marks

Total 15 marks

Question 2

Victoria Jones runs a small business making and selling statues of her cousin the adventurer Tasmania Jones.

The statues are made in a mould, then finished (smoothed and then hand-painted using a special gold paint) by Victoria herself. Victoria sends the statues **in order of completion** to an inspector, who classifies them as either 'Superior' or 'Regular', depending on the quality of their finish.

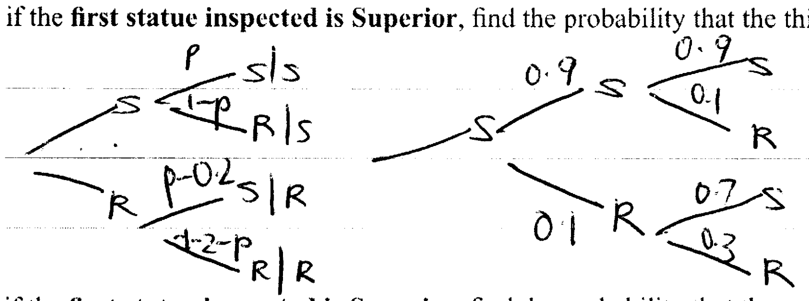
If a statue is Superior then the probability that the next statue completed is Superior is p.

If a statue is Regular then the probability that the next statue completed is Superior is $p - 0.2$.

On a particular day, Victoria knows that $p = 0.9$.

On that day

a. if the **first statue inspected is Superior**, find the probability that the third statue is Regular



$0.9 \times 0.1 + 0.1 \times 0.3$

$= 0.09 + 0.03$

$= 0.12$

2 marks

b. if the **first statue inspected is Superior**, find the probability that the next three statues are Superior

$\xrightarrow{1} S \xrightarrow{0.9} S \xrightarrow{0.9} S \xrightarrow{0.9} S$

$(0.9)^3 = 0.729$

1 mark

c. find the steady state probability that any one of Victoria's statues is Superior.

$$\begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

1 mark

$0.9a + 0.7b = a$

$\therefore 0.9a + 0.7(1-a) = a$

$0.7 - 0.7a = 0.1a$

$0.7 = 0.8a$

$a = \frac{7}{8}$

$\therefore$ Steady state probability that statue is superior $= \frac{7}{8}$

On another day, Victoria finds that if the **first statue inspected is Superior** then the probability that the third statue is Superior is 0.7.

d. **i.** Show that the value of p on this day is 0.75.

$$\begin{bmatrix} p & p-0.2 \\ 1-p & 1.2-p \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{6p}{5} - \frac{1}{5} \\ -\frac{6(p-1)}{5} \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$$

$$\frac{6p}{5} - 0.2 = 0.7$$

$$\frac{6p}{5} = \frac{9}{10}$$

$$\frac{12p}{10} = \frac{9}{10}$$

$$p = \frac{9}{12} = 0.75$$

On this day, a group of 3 consecutive statues is inspected. Victoria knows that the **first** statue of the 3 statues is **Regular**.

ii. Find the expected number of these 3 statues that will be Superior. X = no. of Superiors

X	0	1	2
$Pr(X=x)$	0.2025	0.385	0.4125

$Pr(X=0)$: 3 Regulars $\quad \frac{1}{}R \frac{0.45}{} R \frac{0.45}{} R \quad (0.45)^2 = 0.2025$

$Pr(X=2)$ (1 Regular): $\quad \frac{1}{}R \frac{0.55}{} S \frac{0.75}{} S \quad 0.55 \times 0.75 = 0.4125$

$Pr(X=1)$ (2 Regulars) $\quad \frac{1}{}R \frac{0.55}{} S \frac{0.25}{} R \quad 0.55 \times 0.25 + 0.45 \times 0.55$

$\quad \frac{0.45}{} R \frac{0.55}{} S \quad = 0.385$

$E(X) = 0 \times 0.2025 + 1 \times 0.385 + 2 \times 0.4125 = \boxed{1.21}$

3 + 4 = 7 marks

Victoria hears that another company, Shoddy Ltd, is producing similar statues (also classified as Superior or Regular), but its statues are entirely made by machines, on a construction line. The quality of any one of Shoddy's statues is independent of the quality of any of the others on its construction line. The probability that any one of Shoddy's statues is Regular is 0.8.

Shoddy Ltd wants to ensure that the probability that it produces at least two Superior statues in a day's production run is at least 0.9.

e. Calculate the minimum number of statues that Shoddy would need to produce in a day to achieve this aim.

Let X = no. of Superiors produced in a day

$X \stackrel{d}{=} Bi(p=0.2, n)$

$Pr(X \geq 2) = 0.9$

$\therefore 1 - Pr(X=0) - Pr(X=1) = 0.9$

$\therefore Pr(X=0) + Pr(X=1) = 0.1$

$\therefore {}^nC_0 (0.2)^0 (0.8)^n + {}^nC_1\, 0.2 \times 0.8^{n-1} = 0.1$

$\therefore (0.8)^n + n \times 0.2 \times (0.8)^{n-1} = 0.1$

$n = 17.948$

$\therefore \boxed{n = 18}$

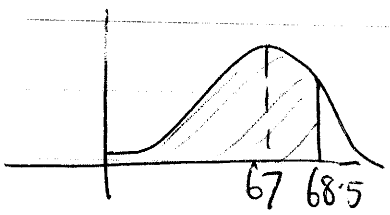
3 marks

Total 14 marks

Question 3

The Bouncy Ball Company (BBC) makes tennis balls whose diameters are normally distributed with mean 67 mm and standard deviation 1 mm. The tennis balls are packed and sold in cylindrical tins that each hold four balls. A tennis ball fits into such a tin if the diameter of the ball is less than 68.5 mm.

a. What is the probability, correct to four decimal places, that a randomly selected tennis ball produced by BBC fits into a tin?



$\Pr(X \leq 68.5) \simeq 0.9332$

$X \stackrel{d}{=} N(\mu = 67, \sigma = 1)$

2 marks

BBC management would like each ball produced to have diameter between 65.6 and 68.4 mm.

b. What is the probability, correct to four decimal places, that the diameter of a randomly selected tennis ball made by BBC is in this range?

$\Pr(65.6 \leq X \leq 68.4) = 0.8385$

2 marks

c. **i.** What is the probability, correct to four decimal places, that the diameter of a tennis ball which fits into a tin is between 65.6 and 68.4 mm?

$\Pr(65.6 \le X \le 68.4 \mid X \le 68.5)$

$= \dfrac{\Pr(65.6 \le X \le 68.4)}{\Pr(X \le 68.5)} = \dfrac{0.838487}{0.933193} = 0.8985$

ii. A tin of four balls is selected at random. What is the probability, correct to four decimal places, that at least one of these balls has diameter outside the desired range of 65.6 to 68.4 mm?

Let Y = no. of balls outside desired range

$Y \overset{d}{=} Bi(n=4,\ p=0.1015)$

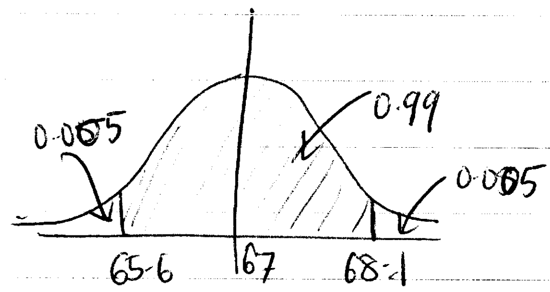
$\Pr(Y \ge 1) = 0.3482$

1 + 2 = 3 marks

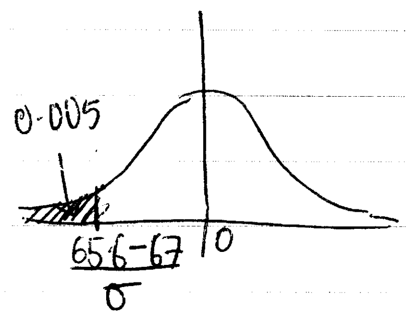
BBC management wants engineers to change the manufacturing process so that 99% of all balls produced have diameter between 65.6 and 68.4 mm. The mean is to stay at 67 mm but the standard deviation is to be changed.

d. What should the new standard deviation be (correct to two decimal places)?

use on CAS InvNorm(0.005, 0, 1)



$\dfrac{-1.4}{\sigma} = -2.57583$



$\therefore \sigma = \dfrac{1.4}{2.57583}$

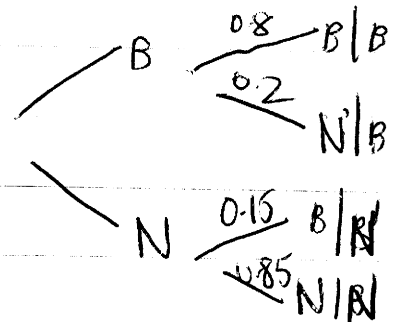
$\sigma \simeq 0.54$ mm

3 marks

BBC sells tennis balls directly to tennis clubs once a year. If a tennis club buys its balls from BBC one year, there is an 80% chance it will buy its balls from BBC the next year. If a tennis club does not buy its balls from BBC one year, there is a 15% chance it will buy its balls from BBC the next year.

Suppose the Melbourne Tennis Club buys its tennis balls from BBC this year.

e. What is the exact probability that it will buy its tennis balls from BBC for the next three years?

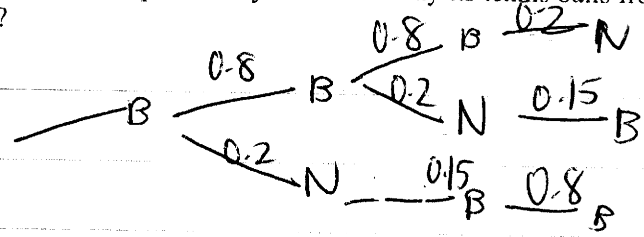


$\xrightarrow{1} B \xrightarrow{0.8} B \xrightarrow{0.8} B \xrightarrow{0.8} B$

$1 \times (0.8)^3 = \frac{64}{125}$

2 marks

f. What is the exact probability that it will buy its tennis balls from BBC for exactly two of the next three years?



$(0.8)^2 \times 0.2$ +

$0.8 \times 0.2 \times 0.15$ +

$0.2 \times 0.15 \times 0.8$ +

$= \boxed{0.176}$ or $\boxed{\frac{22}{125}}$

3 marks

Let p be the probability that the Melbourne Tennis Club will buy its tennis balls from BBC n years after 2009 given that it buys them from BBC in 2009.

g. Find the smallest value of n such that $p \leq 0.45$.

$$\begin{bmatrix} 0.8 & 0.15 \\ 0.2 & 0.85 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} p \\ 1-p \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.15 \\ 0.2 & 0.85 \end{bmatrix}^6 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.47 \\ 0.53 \end{bmatrix}$$

2 marks

Total 17 marks

$$\begin{bmatrix} 0.8 & 0.15 \\ 0.2 & 0.85 \end{bmatrix}^7 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.4566 \\ 0.5434 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.15 \\ 0.2 & 0.85 \end{bmatrix}^8 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.447 \\ 0.553 \end{bmatrix}$$

$\therefore n = 8$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Sharelle is the goal shooter for her netball team. During her matches, she has many attempts at scoring a goal.

Assume that each attempt at scoring a goal is independent of any other attempt. In the long term, her scoring rate has been shown to be 80% (that is, 8 out of 10 attempts to score a goal are successful).

a. **i.** What is the probability, correct to four decimal places, that her first 8 attempts at scoring a goal in a match are successful?

$(0.8)^8 \simeq 0.1678$

ii. What is the probability, correct to four decimal places, that exactly 6 of her first 8 attempts at scoring a goal in a match are successful?

X = no. of successes

$X \stackrel{d}{=} Bi(n=8, p=0.8)$

$Pr(X=6) = 0.2936$

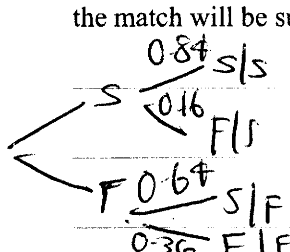
1 + 2 = 3 marks

Assume instead that the success of an attempt to score a goal depends only on the success or otherwise of her previous attempt at scoring a goal.

If an attempt at scoring a goal in a match is successful, then the probability that her next attempt at scoring a goal in the match is successful is 0.84. However, if an attempt at scoring a goal in a match is unsuccessful, then the probability that her next attempt at scoring a goal in the match is successful is 0.64.

Her first attempt at scoring a goal in a match is successful.

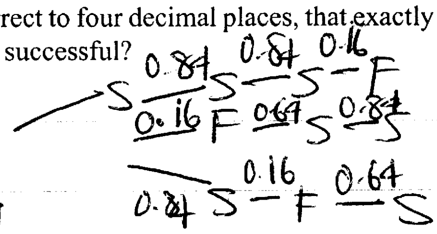
b. **i.** What is the probability, correct to four decimal places, that her next 7 attempts at scoring a goal in the match will be successful?



1 S – S – S – S – S – S – S – S

$(0.84)^7 = 0.2951$

ii. What is the probability, correct to four decimal places, that exactly 2 of her next 3 attempts at scoring a goal in the match will be successful?



$(0.84)^2 \times 0.16 +$

$0.16 \times 0.64 \times 0.84 +$

$0.84 \times 0.16 \times 0.64$

$\boxed{0.2849}$

iii. What is the probability, correct to four decimal places, that her 8th attempt at scoring a goal in the match will be successful?

$$\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}^7 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8000 \\ 0.2000 \end{bmatrix}$$

$\therefore \boxed{0.8000}$

iv. In the long term, what percentage of her attempts at scoring a goal are successful?

$$\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

1 + 3 + 3 + 1 = 8 marks

$0.84a + 0.64b = a$

$0.84a + 0.64(1-a) = a$

$0.64 - 0.64a = 0.16a$

$\therefore 0.64 = 0.8a$

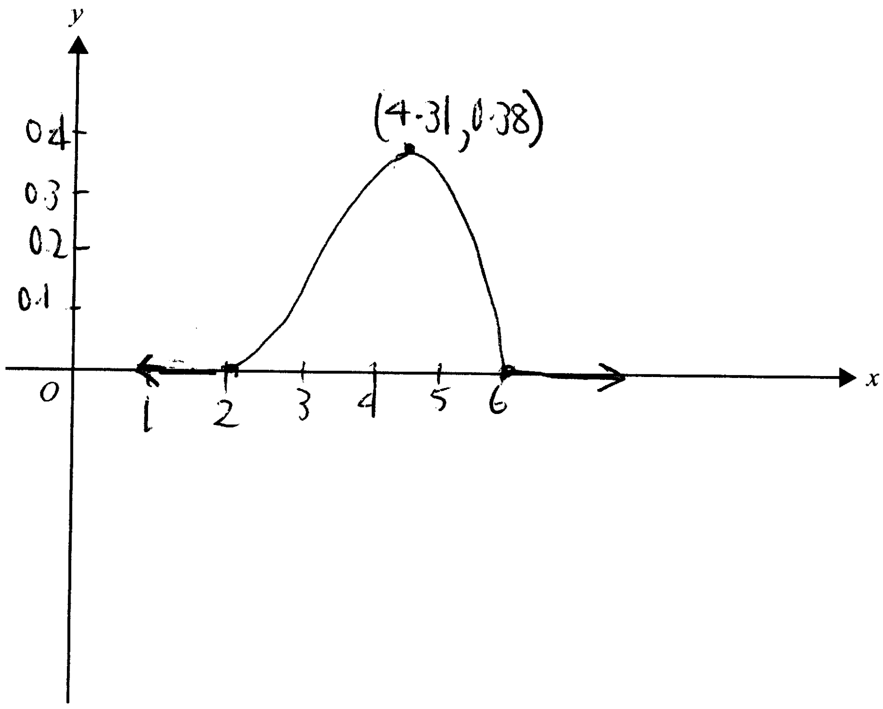
$a = \frac{64}{80} = 0.8$

$\therefore \boxed{80\%}$

The time in hours that Sharelle spends training each day is a continuous random variable with probability density function given by

$$f(x)=\begin{cases}\frac{1}{64}(6-x)(x-2)(x+2) & \text{if } 2\le x\le 6\\ 0 & \text{elsewhere}\end{cases}$$

c. **i.** Sketch the probability density function, and label the local maximum with its coordinates, correct to two decimal places.



ii. What is the probability, correct to four decimal places, that Sharelle spends less than 3 hours training on a particular day?

$$\Pr(X\le 3) = \int_2^3 \frac{1}{64}(6-x)(x-2)(x+2)\,dx$$

$$\simeq 0.1211$$

iii. What is the mean time (in hours), correct to four decimal places, that she spends training each day?

$$E(X) = \int_2^6 \frac{x}{64}(6-x)(x-2)(x+2)\,dx$$

$$= 4.1333$$

2 + 2 + 2 = 6 marks

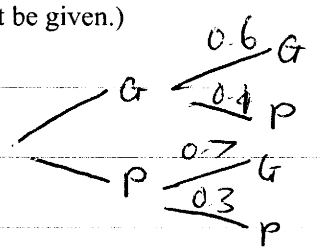
Total 17 marks

Question 2

Each night Kim goes to the gym or the pool. If she goes to the gym one night, the probability she goes to the pool the next night is 0.4, and if she goes to the pool one night, the probability she goes to the gym the next night is 0.7.

a. Suppose she goes to the gym one Monday night.

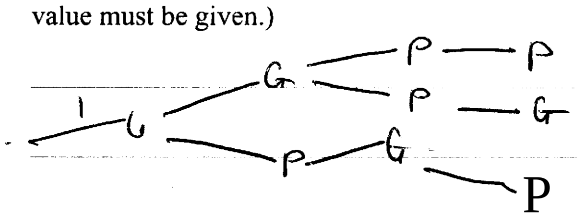
i. What is the probability that she goes to the pool on each of the next three nights? (The exact value must be given.)



$\xrightarrow{1} G \xrightarrow{0.4} P \xrightarrow{0.3} P \xrightarrow{0.3} P$

$0.4 \times 0.3 \times 0.3 = \frac{36}{1000} = \frac{9}{250}$

ii. What is the probability that she goes to the pool on exactly two of the next three nights? (The exact value must be given.)



$0.6 \times 0.4 \times 0.3 +$

$0.6 \times 0.4 \times 0.3 +$

$0.4 \times 0.7 \times 0.4$

$= \boxed{0.408}$

1 + 2 = 3 marks

b. In the long term, what proportion of nights does she go to the pool? (Answer correct to three decimal places.)

$$\begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} x \\ 1-x \end{bmatrix} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$$

$$0.6x + 0.7(1-x) = x$$

$$0.7 - 0.7x = 0.4x$$

$$0.7 = 1.1x$$

$$x = \frac{7}{11}$$

1 mark

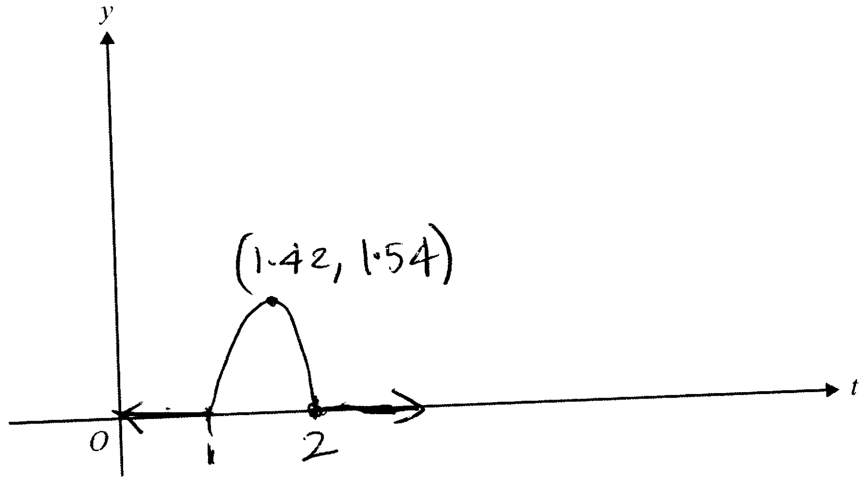
$\therefore$ Long term probability that she goes to pool is

$\frac{4}{11} \approx 0.364$

When Kim goes to the gym, the time, T hours, that she spends working out is a continuous random variable with probability density function given by

$$f(t) = \begin{cases} 4t^3 - 24t^2 + 44t - 24 & \text{if } \quad 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

c. Sketch the graph of $y = f(t)$ on the axes below. Label any stationary points with their coordinates, correct to two decimal places.



3 marks

d. What is the probability, correct to three decimal places, that she spends less than 75 minutes working out when she goes to the gym?

$$\Pr(T \le 1.25) = \int_1^{1.25} (4t^3 - 24t^2 + 44t - 24)\,dt$$

$$\simeq 0.191$$

2 marks

e. What is the probability, correct to two decimal places, that she spends more than 75 minutes working out on 4 out of the 5 next times she goes to the gym?

Let X = no. of times she spends more than 75 min in the gym

p $\quad X \stackrel{d}{=} Bi(n=5, p=0.8086)$

$Pr(X=4) = 0.4091$

≈ 0.41

2 marks

f. Find the median time, to the nearest minute, that she spends working out in the gym.

$\int_{1}^{m} (4t^3 - 24t^2 + 44t - 24)\,dt = 0.5$ where $1 \leq m \leq 2$

Solving: $\therefore m = 1.4588$ hours

≈ 88 minutes

3 marks

Total 14 marks