Tasmania Jones is attempting to recover the lost Zambeji diamond. The diamond is buried at a point 4 km into Death Gorge, which is infested with savage insects. In order to recover the diamond, Tasmania will need to run into the gorge, dig up the diamond and return the same way that he came.

The concentration of insects in the gorge is a **continuous** function of time. The concentration C, insects per square metre, is given by

$$C(t) = \begin{cases} 1000\left(\cos\left(\frac{\pi(t-8)}{2}\right)+2\right)^2 - 1000 & 8 \le t \le 16 \\ m & 0 \le t < 8 \text{ or } 16 < t \le 24 \end{cases}$$

where t is the number of hours after midnight and m is a real constant.

a. What is the value of m?

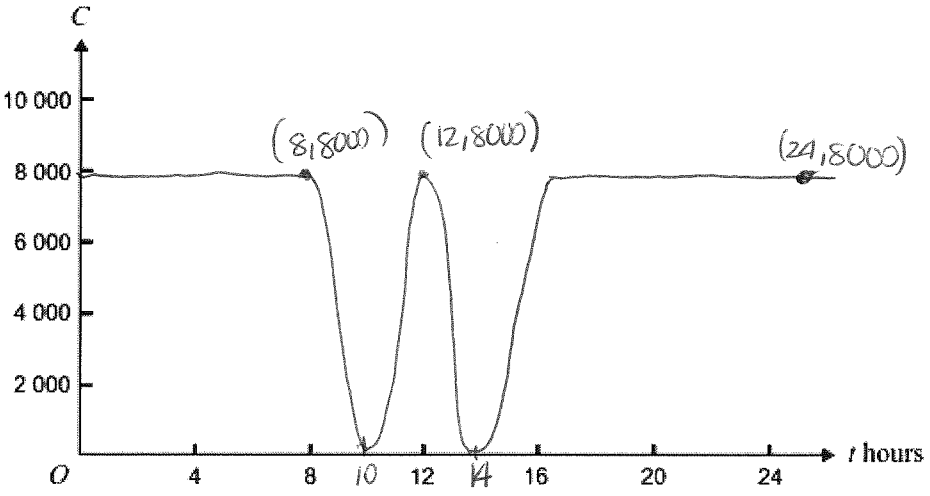
$m = C(16)$

$= 1000\left(\cos\left(\frac{\pi(16-8)}{2}\right)+2\right)^2 - 1000$

$= 8000$

1 mark

b. Sketch the graph of C for $0 \le t \le 24$



3 marks

c. What is the minimum concentration of insects and at what value(s) of t does that occur?

Minimum concentration = 0 at $t = 10, 14$

2 marks

The insects infesting the gorge are known to be deadly if their concentration is more than 1250 insects per square metre.

d. At what time after midnight does the concentration of insects first stop being deadly?

Solving: $C(t) = 1250$ for $8 \leq t \leq 10$

$t = \frac{28}{3}$: at 9:20 AM

1 mark

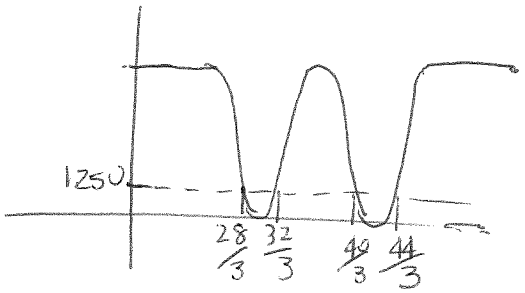
e. During a 24-hour period, what is the total length of time for which the concentration of insects is less than 1250 insects per square metre?

Total time in 24 hours for which $C(t) < 1250$

$= \left(\frac{32}{3} - \frac{28}{3}\right) + \left(\frac{44}{3} - \frac{40}{3}\right)$

$= 2 \times \frac{4}{3} = \frac{8}{3}$ hours

2 marks



Due to the uneven surface of the gorge, the time, T minutes, that Tasmania will take to run x km into the gorge is given by $T = p(q^x - 1)$, where p and q are constants.

f. Tasmania knows that it will take him 5 minutes to run the first kilometre and 12.5 minutes to run the first two kilometres.

i. Find the values of p and q.

$5 = p(q^1 - 1)$ ①

$12.5 = p(q^2 - 1)$ ②

$\therefore \frac{12.5}{5} = \frac{p(q^2-1)}{p(q-1)}$

$\frac{12.5}{5} = \frac{q^2-1}{q-1}$

$\therefore 2.5 = \frac{(q-1)(q+1)}{(q-1)}$

$\therefore 2.5 = q+1$

$\therefore q = 1.5$

$\therefore 5 = p \times 0.5 \quad \therefore p = 10$

ii. Find the length of time that Tasmania will take to run the 4 km to reach the buried diamond.

$T = 10(1.5^x - 1)$

$T(4) = 10(1.5^4 - 1)$

$= \frac{325}{8}$ ~~hours~~ min

2 + 1 = 3 marks

g. Tasmania takes 19 minutes to dig up the diamond and he is able to run back through the gorge in half the time it took him to reach the diamond. Show that it is possible for him to recover the diamond successfully and state how much time he has to spare.

Time taken by Tasmania

$= 40.625 + 19 + 20.3125$ min

$= 79.9375$

Total time available (while concentration is below deadly levels)

$= \frac{4}{3}$ hr

$= 80$ min

$\therefore$ Time to spare

$= (80 - 79.9375)$

$= 0.0625$ min

$= \frac{1}{16}$ min