SOLUTIONS FOR REVISION SHEET (Mr Hanna)

Q1. D

Graph is a sine function translated $\frac{\pi}{4}$ to right

$$\begin{array}{l} -4 \\ c \\ -8 \end{array} \qquad c = \frac{-4 + -8}{2} = -6$$

Period $= \pi$

$\therefore \frac{2\pi}{n} = \pi$

$\therefore n = 2$

$\cancel{a}=$ Amplitude $= 2 \quad \therefore a = 2$

$\therefore y = 2\sin\left(2\left(x - \frac{\pi}{4}\right)\right) - 6$

$\therefore a = 2,\ n = 2,\ \varepsilon = \frac{\pi}{4},\ c = -6$

Q2. A

$\frac{2\pi}{n} = 3\pi \quad \therefore n = \frac{2}{3} \qquad y = a\cos(n(x - h)) + k$

$a = 2$

$h = -\frac{\pi}{6}$

$k = -1$

$\therefore y = 2\cos\left(\frac{2}{3}\left(x + \frac{\pi}{6}\right)\right) - 1$

Q3. E

Amp $= 2$; Not reflected so equation is of form

$y = 2\cos(nx)$. $\quad \frac{2\pi}{n} = 6\pi \quad \therefore n = \frac{1}{3}$

$\therefore y = 2\cos\left(\frac{x}{3}\right) \qquad a = 2,\ n = \frac{1}{3}$

Q4. A

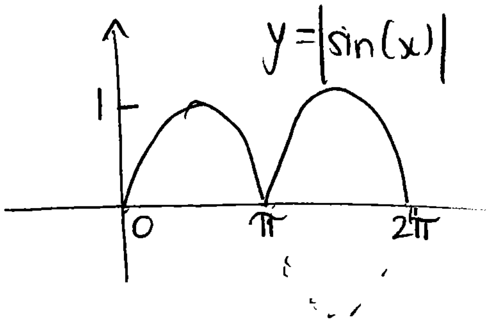
$g(x) = x^2$ so $g\left(\frac{\pi}{3}\right) = \left(\frac{\pi}{3}\right)^2$

$\therefore f(x) = \cos(x)$

so $f\left(g\left(\frac{\pi}{3}\right)\right) = \cos\left(\frac{\pi^2}{9}\right)$

Q5. D

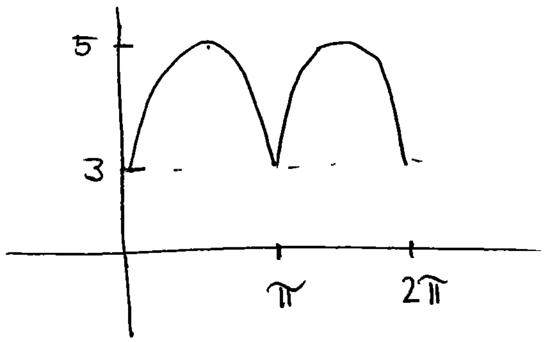
$y = |\sin(x)|$



(or use CAS:

$f_1(x) = 2\,\mathrm{abs}(\sin(x)) + 3)$

$y = 2|\sin(x)| + 3$



Range: $[3, 5]$

Q6. E

$y = 10^{ax} + b$

$\downarrow$

$x = 10^{ay} + b$

$x - b = 10^{ay}$

$ay = \log_{10}(x - b)$

$f^{-1}(x) = \frac{1}{a}\log_{10}(x - b)$

Q7(a)

$$2^{2x} - 6\times 2^{x} + 8 = 0$$

Let $2^{x} = p$

$$p^{2} - 6p + 8 = 0$$

$$(p-4)(p-2) = 0$$

$$p = 4, 2$$

$\therefore 2^{x} = 4$ or $2^{x} = 2$

$$x = 2, 1$$

(b) $5^{2x} - 6\times 5^{x} + 9 = 0$

Let $5^{x} = a$ $\quad a^{2} - 6a + 9 = 0$

$$\therefore (a-3)^{2} = 0$$

$$\therefore a = 3$$

$$\therefore 5^{x} = 3$$

$$\therefore \log_{10} 5^{x} = \log_{10} 3$$

$$x \log_{10} 5 = \log_{10} 3$$

$$x = \frac{\log_{10} 3}{\log_{10} 5}$$

(c) $\log_{10} x + 2\log_{10} 5 = \log_{10} 6$

$$\therefore \log_{10} x + \log_{10} 25 = \log_{10} 6$$

$$\therefore \log_{10}(25x) = \log_{10} 6$$

$$\therefore 25x = 6$$

$$x = \frac{6}{25}$$

Q7. (d) $\log_3(x-a) = b$

$\therefore 3^b = x - a$

$x = a + 3^b$

(e) $2\log_3 x + \log_3 a = 0$

$\therefore \log_3 x^2 + \log_3 a = 0$

$\log_3(ax^2) = 0$

$ax^2 = 3^0$

$\therefore ax^2 = 1$

$x^2 = \frac{1}{a}$

$\therefore x = \sqrt{\frac{1}{a}}$ (since x must be positive, reject $x = -\sqrt{\frac{1}{a}}$)

(f) $2\log_e x + \log_e a = 0$

$\log_e x^2 + \log_e a = 0$

$\log_e(ax^2) = 0$

$ax^2 = e^0$

$x^2 = \frac{1}{a}$

$\therefore x = \sqrt{\frac{1}{a}}$

Q8. B

Eqn of graph shown could be: $y = \log_e(x-2)$

$y = \log_e(x-2)$

$\downarrow$

$x = \log_e(y-2)$

$e^x = y-2$

$y = e^x + 2$

$\therefore f^{-1}(x) = e^x + 2$

Q9.

$\log_{10}(10+5x) - \log_{10}(10-2x) = 1$

$\log_{10}\left(\frac{10+5x}{10-2x}\right) = 1$

$\therefore \frac{10+5x}{10-2x} = 10^1$

$\therefore \frac{10+5x}{10-2x} = 10$

$10+5x = 100-20x$

$25x = 90$

$x = \frac{90}{25}$

$\therefore x = \frac{18}{5}$

Q10.

$f: \mathbb{R} \to \mathbb{R},\ f(x) = e^x$

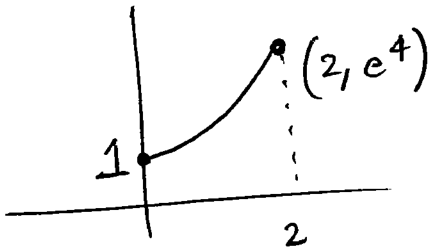
$g: [0,2] \to \mathbb{R},\ g(x) = 2x$

(a) $f(g(x)) = e^{g(x)}$

$= e^{2x}$

$\text{dom}(f(g(x))) = \text{dom}(g) = [0,2]$

$\therefore f(g(x)) = e^{2x},\ x \in [0,2]$



Range: $[1, e^4]$

(b) $(f+g)(x) = 2x + e^x$

$\text{dom}(f+g) = \text{dom}(f) \cap \text{dom}(g)$

$= \mathbb{R} \cap [0,2]$

$= [0,2]$

$\therefore (f+g)(x) = 2x + e^x,\ x \in [0,2]$

Range: $[1, 4+e^2]$

(c) $fg(x) = 2xe^x$

$\text{dom}(fg) = \text{dom}(f) \cap \text{dom}(g)$

$= [0,2]$

Range of $fg = [0, 4e^2]$

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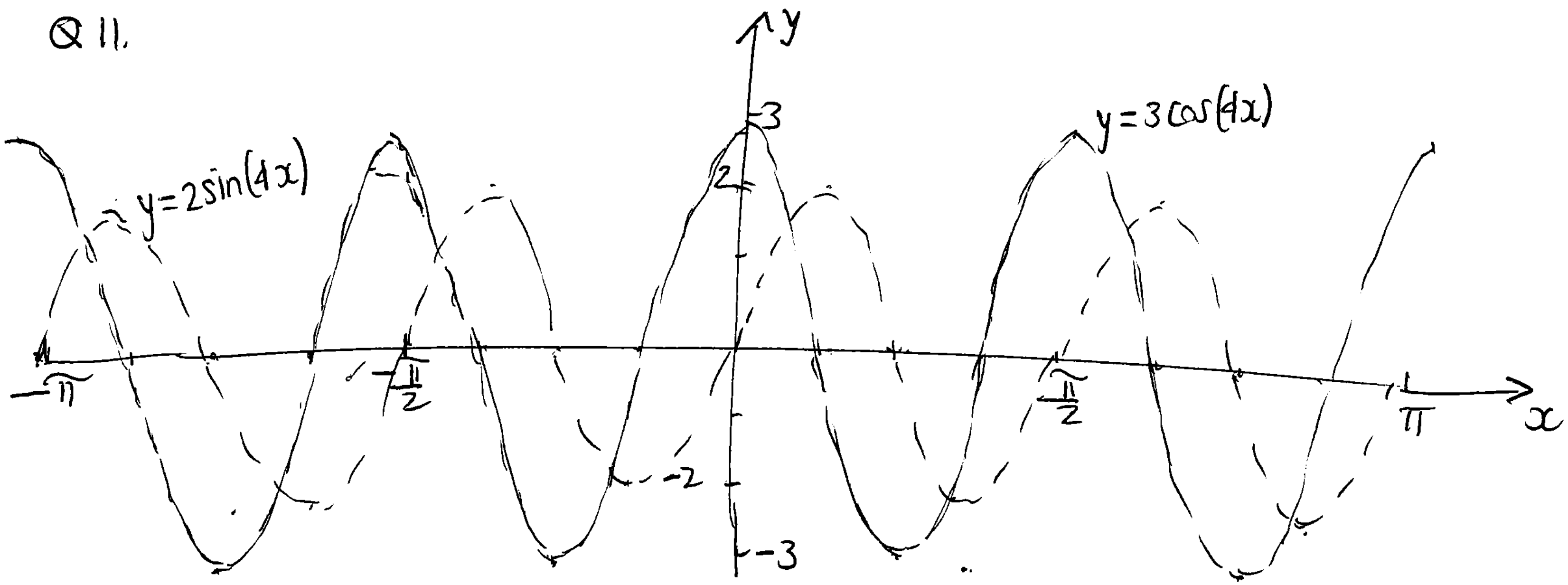
Q 11.



$3\cos(4x) = 2\sin(4x)$ $\qquad -\pi \le x \le \pi$

$\frac{3}{2} = \tan(4x)$ $\qquad -4\pi \le 4x \le 4\pi$

$\therefore$

First solution: $4x = \mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$\therefore x = \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

Period of $\tan(4x) = \frac{\pi}{4}$ $\quad \therefore$ General solution: $x = \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right) + \frac{\pi n}{4}$, $n \in \mathbb{Z}$.

$n=0: x = \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=1: x = \frac{\pi}{4} + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=2: x = \frac{\pi}{2} + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=3: x = \frac{3\pi}{4} + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=-1: x = -\frac{\pi}{4} + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=-2: x = -\frac{\pi}{2} + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=-3: x = -\frac{3\pi}{4} + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

$n=-4: x = -\pi + \frac{1}{4}\mathrm{Tan}^{-1}\left(\frac{3}{2}\right)$

Q12 $f(g(x))$ - is defined when $\text{ran}(g) \subseteq \text{dom}(f)$

$\text{ran}(g) = (0,\infty)$

$\text{dom}(f) = \mathbb{R}$

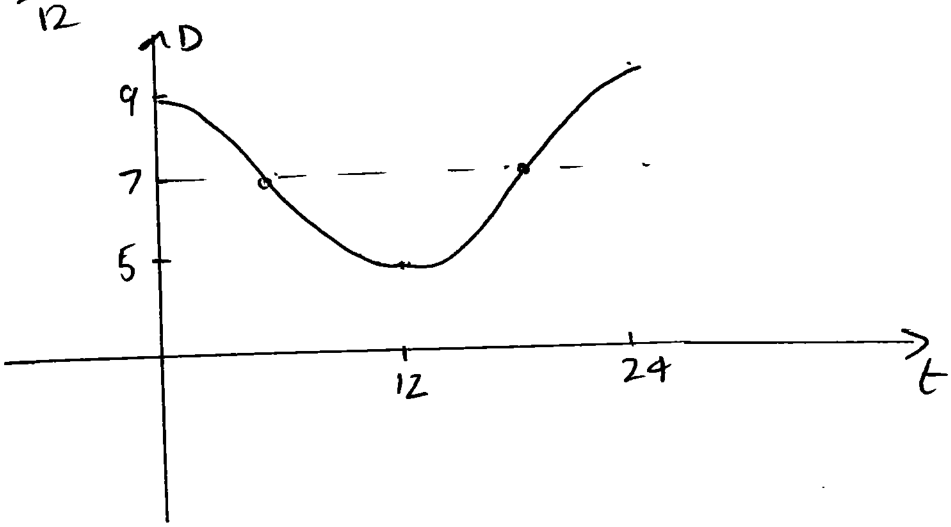
Since $(0,\infty) \subset \mathbb{R}$, $f(g(x))$ is defined.

(b) $f(g(x)) = -2\cos\left(e^{\frac{x}{4}}\right) + 1,\ x \in \mathbb{R}$

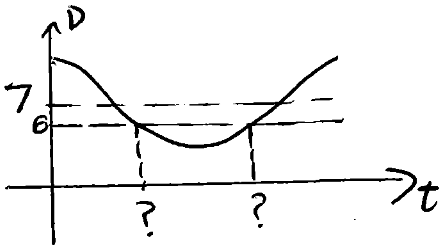
Q13 (a) $D = 18 - 4\cos\left(\frac{\pi t}{12}\right) - \left(11 - 6\cos\left(\frac{\pi t}{12}\right)\right)$

$= 7 + 2\cos\left(\frac{\pi t}{12}\right)$

Period $= \dfrac{2\pi}{\frac{\pi}{12}} = 24$



(b) $D < 6$



Solve $7 + 2\cos\left(\frac{\pi t}{12}\right) = 6$

$\therefore \cos\left(\frac{\pi t}{12}\right) = -\frac{1}{2}$

$\therefore t = 8, 16.$

$\therefore$ Temp difference is less than 6° for $8 < t < 16$.

Q14.

(a) $f(g(x)) = \log_e(2e^{x+1}),\ x \in R$

$= \log_e 2 + \log_e(e^{x+1})$

$= \log_e 2 + (x+1)\log_e e$

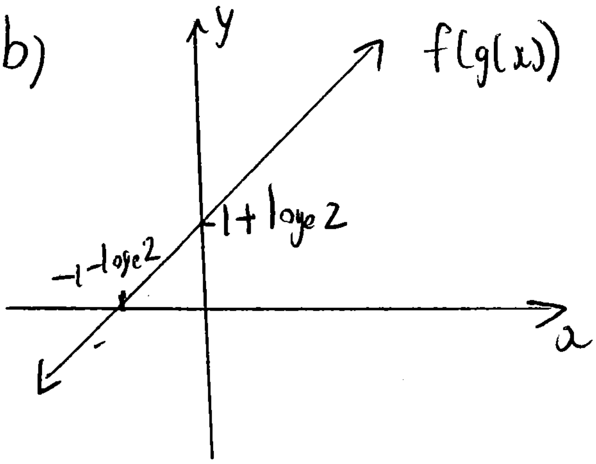
$= x + 1 + \log_e 2,\ x \in R$

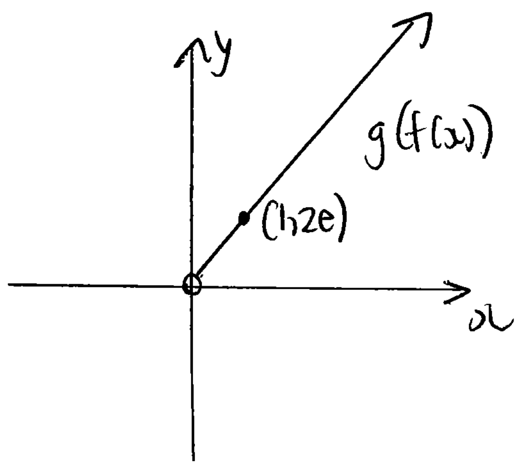
$g(f(x)) = e^{\log_e 2x + 1},\ x > 0$

$= e^1 \cdot e^{\log_e(2x)}$

$= 2ex,\ x > 0.$

(b)





(c) $g(x) = e^{x+1}$

$dom(g)$	$ran(g)$
R	$(0,\infty)$
$dom(g^{-1})$	$ran(g)$
$(0,\infty)$	R

$x = e^{y+1}$

$y + 1 = \log_e x$

$\therefore g^{-1}(x) = \log_e x - 1,$

$x \in (0,\infty)$