SOLUTIONS.

Core

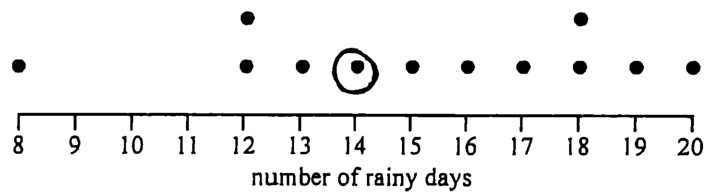
Question 1

Table 1 shows the number of rainy days recorded in a high rainfall area for each month during 2008.

Table 1

Month	Number of rainy days
January	12
February	8
March	12
April	14
May	18
June	18
July	20
August	19
September	17
October	16
November	15
December	13

The dot plot below displays the distribution of the number of rainy days for the 12 months of 2008.



a. Circle the dot on the dot plot that represents the number of rainy days in April 2008.

1 mark

b. For the year 2008, determine

i. the median number of rainy days per month

$n=12$ Position of median: $\left(\frac{12+1}{2}\right)$th $= 6\frac{1}{2}$th position → Median $= \frac{(15+16)}{2} = 15.5$

ii. the percentage of months that have more than 10 rainy days. Write your answer correct to the nearest per cent.

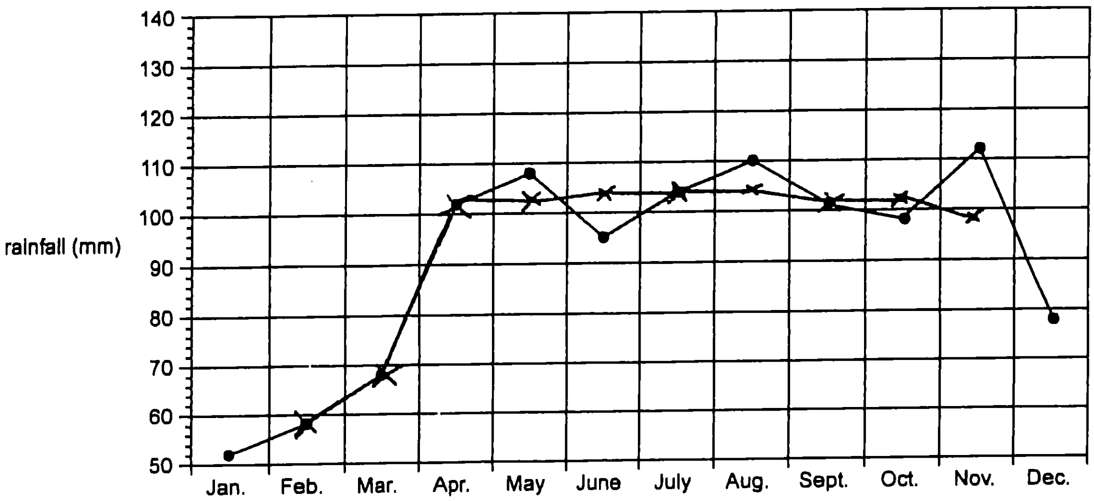
$\frac{11}{12} \times 100 = 92\%$

1 + 1 = 2 marks

Core – continued

Question 2

The time series plot below shows the rainfall (in mm) for each month during 2008.



a. Which month had the highest rainfall?

November

1 mark

b. Use three-median smoothing to smooth the time series. Plot the smoothed time series on the plot above. Mark each smoothed data point with a cross (×).

2 marks

c. Describe the general pattern in rainfall that is revealed by the smoothed time series plot.

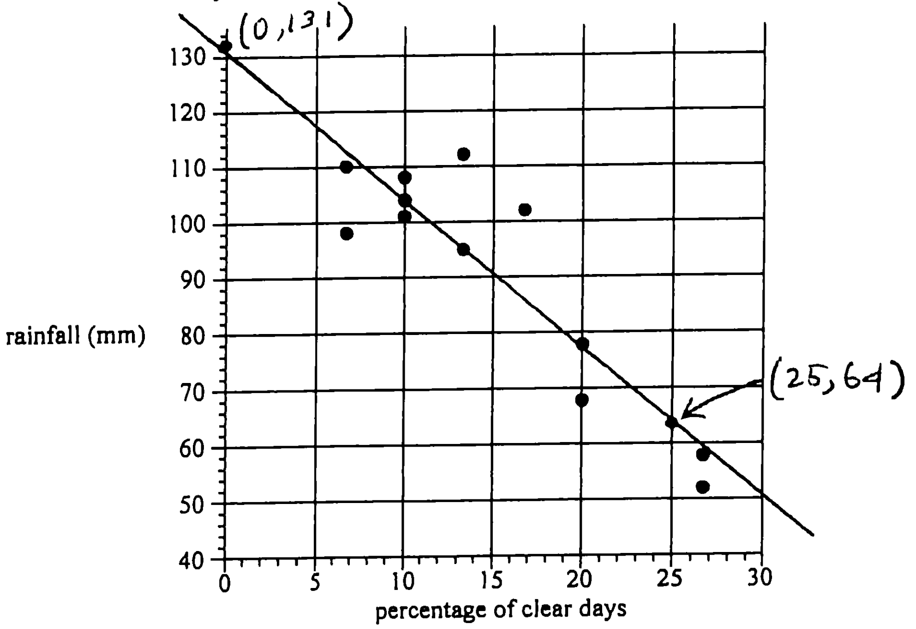
After an initially increasing trend for the first three months, the overall trend from April to November was for the rainfall to remain relatively constant

1 mark

Question 3

The scatterplot below shows the *rainfall* (in mm) and the *percentage of clear days* for each month of 2008.

Co-ordinates:
(0, 131)
(25, 64)



An equation of the least squares regression line for this data set is

$$rainfall = 131 - 2.68 \times percentage\ of\ clear\ days$$

a. Draw this line on the scatterplot.

1 mark

b. Use the equation of the least squares regression line to predict the rainfall for a month with 35% of clear days. Write your answer in mm correct to one decimal place.

Rainfall = 131 − 2.68 × 35

= 37.2 mm

1 mark

c. The coefficient of determination for this data set is 0.8081.

i. Interpret the coefficient of determination in terms of the variables *rainfall* and *percentage of clear days*.

80.81% of the variation in rainfall can be explained by the variation in percentage of clear days.

ii. Determine the value of Pearson's product moment correlation coefficient. Write your answer correct to three decimal places.

$r = -\sqrt{0.8081}$

1 + 2 = 3 marks

$= -0.899$

Question 4

a. Table 2 shows the seasonal indices for rainfall in summer, autumn and winter. Complete the table by calculating the seasonal index for spring.

Table 2

Seasonal indices			
summer	autumn	winter	spring
0.78	1.05	1.07	1.1

Index for spring = $4 - (0.78 + 1.05 + 1.07) = 1.1$

1 mark

b. In 2008, a total of 188 mm of rain fell during summer.

Using the appropriate seasonal index in Table 2, determine the deseasonalised value for the summer rainfall in 2008. Write your answer correct to the nearest millimetre.

Deseasonalized value = $\frac{\text{actual value}}{\text{index}}$

Deseasonalized summer value = $\frac{188}{0.78} = 241$ mm

1 mark

c. What does a seasonal index of 1.05 tell us about the rainfall in autumn?

It is 5% above the yearly average for the four seasons.

1 mark

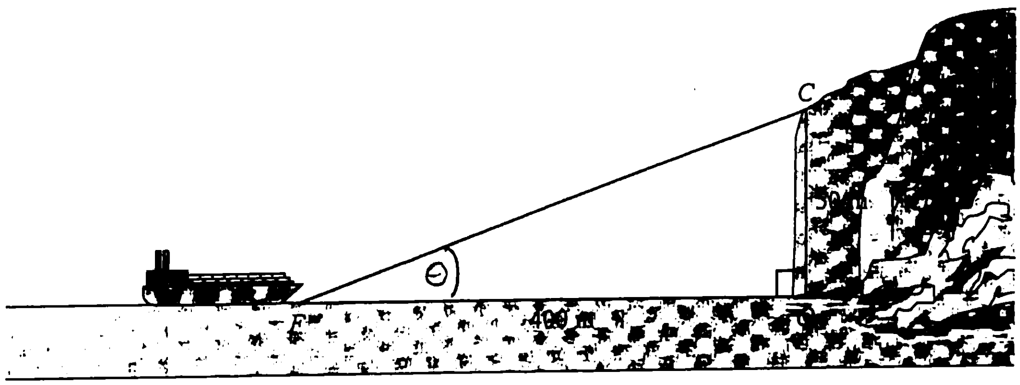
Total 15 marks

END OF CORE
TURN OVER

Module 2: Geometry and trigonometry

Question 1

A ferry, F, is 400 metres from point O at the base of a 50 metre high cliff, OC.



a. Show that the gradient of the line FC in the diagram is 0.125.

$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{50}{400} = \frac{1}{8} = 0{\cdot}125$

1 mark

b. Calculate the angle of elevation of point C from F.

Write your answer in degrees, correct to one decimal place.

$\tan(\theta) = 0{\cdot}125$

$\theta = \tan^{-1}(0{\cdot}125) = 7{\cdot}1^{\circ}$

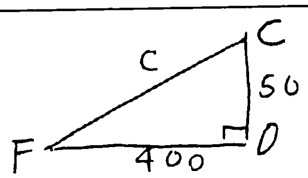
1 mark

c. Calculate the distance FC, in metres, correct to one decimal place.

$c^2 = 50^2 + 400^2$

$c = \sqrt{50^2 + 400^2}$

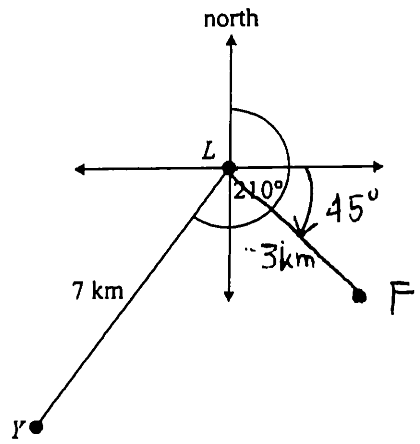
1 mark



$C = 403{\cdot}1$ m.

Question 2

A yacht, Y, is 7 km from a lighthouse, L, on a bearing of 210° as shown in the diagram below.



a. A ferry can also be seen from the lighthouse. The ferry is 3 km from L on a bearing of 135°. On the diagram above, label the position of the ferry, F, and show an angle to indicate its bearing.

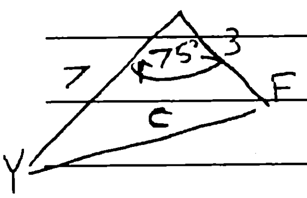
1 mark

b. Determine the angle between LY and LF.

$210° - 135° = 75°$

1 mark

c. Calculate the distance, in km, between the ferry and the yacht correct to two decimal places.



$c^2 = 7^2 + 3^2 - 2 \times 7 \times 3 \cos(75°)$

$c^2 = 47.1296...$

$c = \sqrt{47.1296...}$ Distance = 6.87 km

$c = 6.87$

1 mark

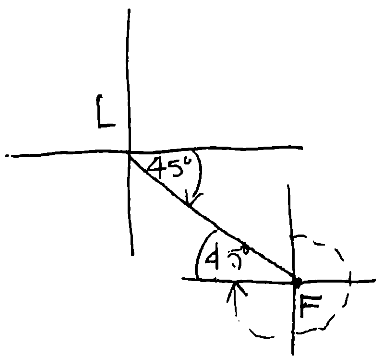
d. Determine the bearing of the lighthouse from the ferry.

Bearing of L from F

$= 270° + 45°$

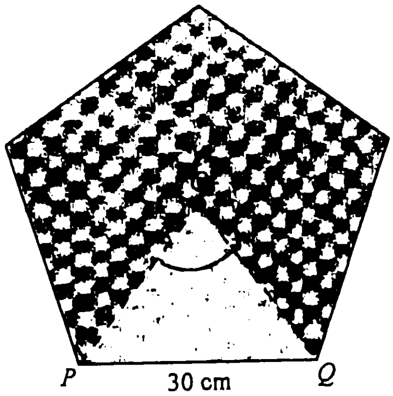
$= 315°$ T.

1 mark



Question 3

The ferry has a logo painted on its side. The logo is a regular pentagon with centre O and side length 30 cm. It is shown in the diagram below.

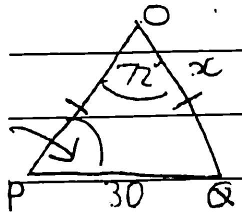


a. Show that angle POQ is equal to 72°.

$$\angle POQ = \frac{360}{5}$$
$$= 72^{\circ}$$

1 mark

b. Show that, correct to two decimal places, the length OP is 25.52 cm $\triangle OPQ$ is isosceles.

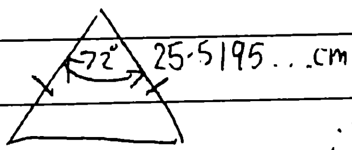


$(180-72)/2$
$= 54°$

$$\frac{x}{\sin(54)} = \frac{30}{\sin(72)}$$
$$x = \frac{30\sin(54)}{\sin(72)} = 25.52 \text{ cm}$$

1 mark

c. Find the area of the pentagon. Write your answer correct to the nearest cm².



$$A = \frac{1}{2} \times (25.5195\ldots)^2 \times \sin(72^{\circ})$$
$$= 309.6859\ldots \text{ cm}^2$$

$\therefore$ Total area of logo (5 triangles)

$$= 5 \times 309.6859\ldots$$
$$= 1548.43 \text{ cm}^2$$
$$= 1548 \text{ cm}^2$$

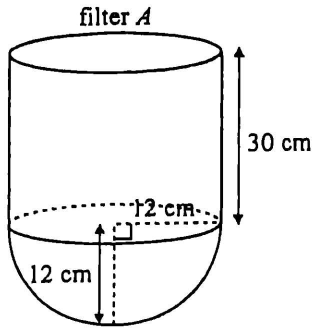
2 marks

Question 4

The ferry has two fuel filters, A and B.

Filter A has a hemispherical base with radius 12 cm.

A cylinder of height 30 cm sits on top of this base.



a. Calculate the volume of filter A. Write your answer correct to the nearest cm^3.

$V = V_{hemisphere} + V_{cylinder}$

$= \frac{2\pi r^3}{3} + \pi r^2 h$

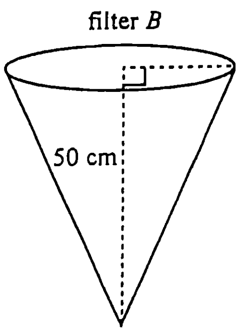
$= \frac{2\pi}{3} \times 12^3 + \pi \times 12^2 \times 30$

$= 17190.795 \text{ cm}^3$

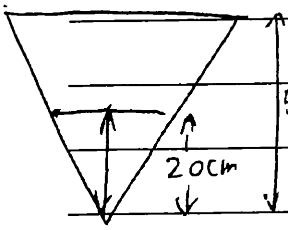
$= 17191 \text{ cm}^3$

2 marks

Filter B is a right cone with height 50 cm.



b. Originally filter B was full of oil, but some was removed.
If the height of the oil in the cone is now 20 cm, what percentage of the original volume of oil was removed?



	Small	Big
k	20 :	50
	= 2 :	5
k^3	2^3 :	5^3
	= 8 :	125

Percentage of oil removed = 93.6%

2 marks

Percentage of oil remaining = $\frac{8}{125} \times 100$

$= 6.4\%$

Percentage removed $= 93.6\%$

Total 15 marks

END OF MODULE 2
TURN OVER

Module 6: Matrices

Question 1

Three types of cheese, Cheddar (C), Gouda (G) and Blue (B), will be bought for a school function.
The cost matrix P lists the prices of these cheeses, in dollars, at two stores, Foodway and Safeworth.

$$P = \begin{array}{c} \begin{bmatrix} 6.80 & 5.30 & 6.20 \\ 7.30 & 4.90 & 6.15 \end{bmatrix} \end{array} \begin{array}{l} \textit{Foodway} \\ \textit{Safeworth} \end{array}$$

a. What is the order of matrix P?

2×3

1 mark

The number of packets of each type of cheese needed is listed in the quantity matrix Q.

$$Q = \begin{bmatrix} 8 \\ 11 \\ 3 \end{bmatrix} \begin{array}{l} C \\ G \\ B \end{array}$$

b. i. Evaluate the matrix $W = PQ$.

$$\underset{2\times 3}{\begin{array}{c} F \\ S \end{array} \begin{bmatrix} 6.80 & 5.30 & 6.20 \\ 7.30 & 4.90 & 6.15 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 8 \\ 11 \\ 3 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 6.8 \times 8 + 5.30 \times 11 + 6.20 \times 3 \\ 7.30 \times 8 + 4.90 \times 11 + 6.15 \times 3 \end{bmatrix}}$$

$$= \begin{array}{c} F \\ S \end{array} \begin{bmatrix} 131.3 \\ 130.75 \end{bmatrix}$$

ii. At which store will the total cost of the cheese be lower?

Safeworth

1 + 1 = 2 marks

Question 2

Tickets for the function are sold at the school office, the function hall and online.

Different prices are charged for students, teachers and parents.

Table 1 shows the number of tickets sold at each place and the total value of sales.

Table 1

	School office	**Function hall**	**Online**
Student tickets	283	35	84
Teacher tickets	28	4	3
Parent tickets	5	2	7
Total sales	\$8712	\$1143	\$2609

For this function

- student tickets cost $\$x$
- teacher tickets cost $\$y$
- parent tickets cost $\$z$.

a. Use the information in Table 1 to complete the following matrix equation by inserting the missing values in the shaded boxes.

$$A \cdot X = K$$

$$\begin{bmatrix} 283 & 28 & 5 \\ 35 & 4 & 2 \\ 84 & 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix}$$

1 mark

b. Use the matrix equation to find the cost of a teacher ticket to the school function.

$$X = A^{-1} K$$

$$X = \begin{bmatrix} 283 & 28 & 5 \\ 35 & 4 & 2 \\ 84 & 3 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 27 \\ 32 \\ 35 \end{bmatrix}$$

$y = 32$

Cost of a teacher's ticket = \$32

2 marks

Question 3

In 2009, the school entered a Rock Eisteddfod competition.

When rehearsals commenced in February, all students were asked whether they thought the school would make the state finals. The students' responses, 'yes', 'no' or 'undecided' are shown in the initial state matrix S_0.

$$S_0 = \begin{array}{c} \begin{bmatrix} 160 \\ 120 \\ 220 \end{bmatrix} \end{array} \begin{array}{l} \textit{yes} \\ \textit{no} \\ \textit{undecided} \end{array}$$

a. How many students attend this school?

$160 + 120 + 220 = 500$

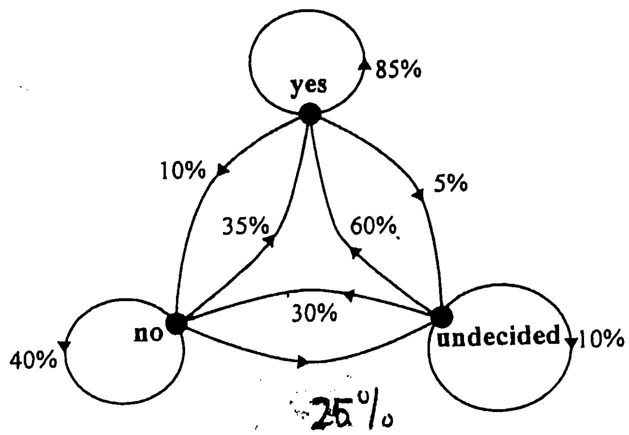
1 mark

Each week some students are expected to change their responses. The changes in their responses from one week to the next are modelled by the transition matrix T shown below.

$$\begin{array}{c} \textit{response this week} \\ \begin{array}{ccc} \textit{yes} & \textit{no} & \textit{undecided} \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.85 & 0.35 & 0.60 \\ 0.10 & 0.40 & 0.30 \\ 0.05 & 0.25 & 0.10 \end{bmatrix} \begin{array}{l} \textit{yes} \\ \textit{no} \\ \textit{undecided} \end{array} \quad \begin{array}{l} \textit{response} \\ \textit{next week} \end{array}$$

The following diagram can also be used to display the information represented in the transition matrix T.



b. **i.** Complete the diagram above by writing the missing percentage in the shaded box.

ii. Of the students who respond 'yes' one week, what percentage are expected to respond 'undecided' the next week when asked whether they think the school will make the state finals?

5%

iii. In total, how many students are **not** expected to have changed their response at the end of the first week?

$0.85 \times 160 + 0.40 \times 120 + 0.10 \times 220$

$= 206$

1 + 1 + 2 = 4 marks

c. Evaluate the product $S_1 = T S_0$, where S_1 is the state matrix at the end of the first week.

$$S_1 = \begin{bmatrix} 0.85 & 0.35 & 0.60 \\ 0.10 & 0.40 & 0.30 \\ 0.05 & 0.25 & 0.10 \end{bmatrix} \begin{bmatrix} 160 \\ 120 \\ 220 \end{bmatrix}$$

$$= \begin{bmatrix} 310 \\ 130 \\ 60 \end{bmatrix}$$

1 mark

d. How many students are expected to respond 'yes' at the end of the third week when asked whether they think the school will make the state finals?

$S_3 = T^3 S_0 = \begin{bmatrix} 361 \\ 91.1 \\ 47.9 \end{bmatrix}$ 361 are expected to respond 'yes'.

1 mark

Question 4

A series of extra rehearsals commenced in April. Each week participants could choose extra dancing rehearsals or extra singing rehearsals.

A matrix equation used to determine the number of students expected to attend these extra rehearsals is given by

$$L_{n+1} = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \times L_n - \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

where L_n is the column matrix that lists the number of students attending in week n.

The attendance matrix for the first week of extra rehearsals is given by

$$L_1 = \begin{bmatrix} 95 \\ 97 \end{bmatrix} \begin{matrix} \textit{dancing} \\ \textit{singing} \end{matrix}$$

a. Calculate the number of students who are expected to attend the extra singing rehearsals in week 3.

$L_2 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 95 \\ 97 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix}$ 68 will be

$L_3 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 100 \\ 80 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 68 \end{bmatrix}$ expected.

1 mark

b. Of the students who attended extra rehearsals in week 3, how many are not expected to return for any extra rehearsals in week 4?

12

1 mark

Total 15 marks

END OF QUESTION AND ANSWER BOOK