ANSWERS.

Finance Maths Practice Test 2016

Question 1

$P_0 = 2000, P_{n+1} = 1.5P_n - 500$

The first three terms generated by this recurrence relation would be:

A. 500, 2500, 2000 ...
B. 2000, 1500, 1000...
C. 2000, 2500, 3000...
D. 2000, 2500, 3250....
E. 2000, 3000, 4500...

$P_1 = 1.5 \times 2000 - 500 = 2500$

$P_2 = 1.5 \times 2500 - 500 = 3250$

Question 2

Which of the following recurrence relations will generate a sequence that decays geometrically?

A. $L_0 = 2000, L_{n+1} = L_n - 150$
B. $L_0 = 2000, L_{n+1} = L_n + 150$
C. $L_0 = 2000, L_{n+1} = 0.65L_n$
D. $L_0 = 2000, L_{n+1} = 6.5L_n$
E. $L_0 = 2000, L_{n+1} = 0.85L_n - 100$

Question 3

Eva has $1200 that she plans to invest for one year.
One company offers to pay her interest at the rate of 6.75% per annum compounding daily.
The effective annual interest rate would be closest to:

A. 6.75%
B. 6.92%
C. 6.96%
D. 6.98%
E. 6.99%

$\text{eff}(6.75, 365) \approx 6.98$

Question 5

To purchase a house Sam has borrowed $250 000 at an interest rate of 4.45% per annum, fixed for ten years. Interest is calculated monthly on the reducing balance of the loan. Monthly repayments are set at $1382.50. After 10 years, Sam renegotiates the conditions for the balance of his loan. The new interest rate will be 4.25% per annum. He will pay $1750 per month. The total time it will take him to pay out the loan fully is closest to

A. 17 years.

B. 20 years.

C. 21 years.

D. 22 years.

E. 23 years.

N = 120
I = 4.5
PV = 250000
FV = ?
PMT = -1382.50
P/Y = 12
C/Y = 12
gives FV = -182716.86

N = ?
I = 4.25
PV = 182716.86
FV = 0
PMT = -1750
P/Y = 12
C/Y = 12 gives
N = 130.6

Total time: 10 years + $\frac{131}{12} \approx 21$ yrs

Question 6

Leslie borrowed $35 000 from a bank. Interest is charged at the rate of 4.75% on the reducing monthly balance. The loan is to be repaid with 47 monthly payments of $802.00 and a fi nal payment that is to be adjusted so that the loan will be fully repaid after exactly 48 monthly payments. Correct to the nearest cent, the amount of the final payment will be

A. $0.39
B. $3.57
C. $802.00
D. $802.39
E. $805.57

N = 47
I = 4.75
PV = 35000
PMT = -802
FV = ?
P/Y = 12
C/Y = 12
gives
FV = -802.39

Interest charged in last month
$= 802.39 \times \frac{4.75/12}{100} = 3.18$

∴ Total payment
$= 802.39 + 3.18 = \$805.57$

Question7

Cindy took out a reducing balance loan of $8400 to finance an overseas holiday. Interest was charged at a rate of 9% per annum, compounding quarterly. Her loan is to be fully repaid in six years, with equal quarterly payments. After three years, Cindy will have reduced the balance of her loan by approximately:

A. 9%
B. 35%
C. 43%
D. 50%
E. 57%

N = 24
I = 9
PV = 8400
PMT = ?
FV = 0
P/Y = 4
C/Y = 4
gives PMT = -456.79

N = 12
I = 9
PMT = -456.79
FV = ?
P/Y = 4
C/Y = 4
PV = 8400
gives
FV = -4757.41

Since she still owes $4757, she has reduced balance by:
8400 - 4757 = 3643
$\frac{3643}{8400} \times \frac{100}{1} \approx 43\%$

Question 8

Anthony invested \$15,000 in an account. It earned r % per annum, compounding monthly, The amount of interest that is earned in the third year of the investment is

A. $15000\left(1+\frac{r}{1200}\right)^3 - 15000\left(1+\frac{r}{1200}\right)^2$

B. $15000\left(1+\frac{r}{1200}\right)^{36} - 15000\left(1+\frac{r}{1200}\right)^{24}$

C. $15000\left(1+\frac{r}{100}\right)^3 - 15000\left(1+\frac{r}{100}\right)^2$

D. $15000\left(1+\frac{r}{100}\right)^{36} - 15000\left(1+\frac{r}{100}\right)^{24}$

E. $15000\left(1+\frac{r}{1200}\right)^4 - 15000\left(1+\frac{r}{1200}\right)^3$

$V_{n+1} = RV_n \qquad R = 1 + \frac{r/m}{100}$

$\therefore R = 1 + \frac{r/12}{100}$

$= 1 + \frac{r}{1200}$

$\therefore V_{n+1} = \left(1 + \frac{r}{1200}\right) V_n$

$\therefore V_n = 15000\left(1 + \frac{r}{1200}\right)^n$ where n = no. of months

Interest earned in 3rd year

$= V_{36} - V_{24}$

$= 15000\left(1 + \frac{r}{1200}\right)^{36} - 15000\left(1 + \frac{r}{1200}\right)^{24}$

Question 1

Harry won $6,000 in a marathon race and deposited this money into a savings account. The value of Harry's savings, after n months, can be modelled by the recurrence relation below:

$$S_0 = 6000, S_{n+1} = 1.003S_n$$

a. What is the annual interest rate, (compounding monthly) for Harry's savings account?

Monthly rate = $0.003 \times 100 = 0.3\%$

∴ Annual rate = $12 \times 0.3\%$

$= 3.6\%$

1 mark

b. What would be the value of Harry's savings after 18 months? Give your answer to the nearest dollar.

$S_n = 6000 \times 1.003^n$

$S_{18} = 6000 \times 1.003^{18} = \$10215$

1 mark

Using a different investment strategy, Harry could deposit $4000 in an investment account earnng 4.2% per annum compounding monthly and make additional payments of $250 at the end of every month. Let T_n be the balance in Harry's account after n months using this strategy.

c. Write down a recurrence relation in terms of T_n and T_{n+1} which models the value of this investment using this strategy

$R = 1 + \frac{r/m}{100} = 1 + \frac{4.2/12}{100} = 1.0035$

$\therefore T_{n+1} = 1.0035\, T_n + 250$

$T_0 = 4000$

1mark

d. What is the total interest that Harry would have earned after six months? Give your answer to the nearest cent.

N = 6
I = 4.2
PV = −4000
PMT = −250
FV = ?
P/Y = 12
C/Y = 12
gives FV = 5597.92

Interest earned
$= 5597.92 - 4000 - 6 \times 250$
$= \$97.92$

2 marks

Question 2

Tim needs to buy a new stereo system. He borrowed $7500 to pay for it and will be charged interest at the rate of 5.76% per annum, compounding monthly. Hugo will fully repay this loan with repayments of $430 each month.

a. How many repayments are required to fully repay this loan? Round your answer to the nearest whole number.

N = ?
I = 5·76
PMT = −430
FV = 0
P/Y = 12
C/Y = 12

gives N = 18·26
∴ ≈ 18 payments

1 mark

After the fifth repayment, Tim increased his monthly repayment so that the loan was fully repaid with a further seven repayments (that is, 12 repayments in total)

. b. What is the minimum value of Tim's new monthly repayment? Give your answer to the nearest cent.

N = 5
I = 5·76
PMT = −430
PV = 7500
FV = ?
P/Y = 12
C/Y = 12 gives FV = −5511·00

N = 7
I = 5·76
PMT = ?
PV = 5511·00
FV = 0
P/Y = 12
C/Y = 12

gives PMT = −802·47

New payment = $802·47

2 marks

Question 3

A sponsor of a cricket club has invested $20,000 in a perpetuity. The annual interest from this perpetuity is $750. The interest from the perpetuity is given to the best player in the club every year, for a period of 10 years.

a. What is the annual rate of interest for this perpetuity investment?

$D = \frac{r/m}{100} \times V_0$ r = ? m = 1 V_0 = 20000 D = 750

$750 = \frac{r}{100} \times 20000$

r = 3·75% p.a.

1 mark

b. After 10 years, how much money is still invested in the perpetuity?

$20,000

1 mark

Question 4

Gregory is learning to be a pilot and is lucky enough to fly in a brand new Jabiru J170-D aircraft for his lessons. The value of this aircraft will be depreciated using a flat-rate depreciation method. A recurrence relation that models the value of the aircraft after n years, V*n*, is shown below.

$V_0 = 90\,000$, $V_{n+1} = V_n - 4000$

a What was the purchase price of the aircraft?

$V_0 = 90000 \quad \therefore \$90,000$

1 mark

b What is the annual percentage rate of depreciation for the aircraft? Write your answer correct to one decimal place.

$\frac{4000}{90000} \times \frac{100}{1} = 4.4\%$ p.a.

1 mark

c The aircraft will be sold when its value first drops below $60 000.

i After how many years will this aircraft be sold?

$V_n = 90000 - 4000n$

$60000 = 90000 - 4000n$ at end of

$n = 7.5 \quad \therefore$ Will be sold 1 mark

8th year.

ii What price would it be sold for?

$90000 - 4000 \times 8$

$= \$58,000$

1 mark

The value of the aircraft actually depends on the number of hours it has flown rather than it's age. The Freewings Flying Club estimates that the Jabiru J170-D will fly an average of 5 hours per week. After one year (52 weeks), the value of the aircraft will be $83 500.

d i Calculate the depreciation in the value of the aircraft after 1 year.

$90000 - 83500 = \$6500$

1 mark

Ii. Calculate the depreciation of the aircraft per hour it has flown.

Hours flown in 1 week = 5

$\therefore$ Hours flown in year $= 52 \times 5 = 260$

$6500 \div 260 = \$25$ 1 mark

$\therefore \$25$ / hour

Question

Michelle purchased a $17 000 car. The car's value depreciates at the rate of 10% per annum using the reducing balance method.

a. Write down a rule which gives the value V_n of the car after n years.

$$V_n = \left(1 - \frac{10}{100}\right)^n \times 17000$$

$$\therefore V_n = (0.9)^n \times 17000$$

1 mark

b. By what amount, in dollars, does the car's value depreciate during Michelle's third year of ownership

$V_2 = (0.9)^2 \times 17000 = 13770$

$V_3 = (0.9)^3 \times 17000 = 12393$

$V_2 - V_3 = 13770 - 12393$

$= \$1377$

2 marks

c. After how many years of ownership will the car's value fi rst be below $7000?

$0.9^n \times 17000 = 7000$

Solving: $n = 8.42$

$\therefore$ During 9 th year.

1 mark

Question

Instead of going on long service leave, Peter instead opts to take a lump sum payment of $40000 which he invests in annuity that guarantees a stable interest rate of 4.5% per annum compounding fortnightly. Peter receives fortnightly payments of $300 from the annuity.

a. How many **full** payments of $300 will Peter receive?

N = ?
I = 4.5
PV = −40000
PMT = 300
FV = 0
P/Y = 26
C/Y = 26

gives N = 151.72

$\therefore$ Receives 151 full payments

b. The final payment needs to be adjusted. Calculate the final payment that Peter will receive from the annuity, correct to the nearest dollar.

N = 151
I = 4.5
PV = −40000
PMT = 300
FV = ?
P/Y = 26
C/Y = 26

gives FV = 215.50

Interest credited on $215.50

$= 215.50 \times \frac{\frac{4.5}{26}}{100}$

$= 0.37$

$\therefore$ Total payment = $215.87

c. What is the total interest that Peter received from the annuity?

Total paid out $= 151 \times 300 + 215.87$

$\therefore$ Interest $= 151 \times 300 + 215.87 - 40000$

$= \$5515.87$

$\approx \$5516$