Question 3

A function f is defined by the rule $f(x) = \log_e(5 - x) + 1$.

a. Sketch the graph of f over its maximal domain on the axes below. Clearly label any intersections **with the axes with their exact coordinates** and any asymptotes **with their equations**.

x-int:

$\log_e(5 - x) + 1 = 0$

$\log_e(5 - x) = -1$

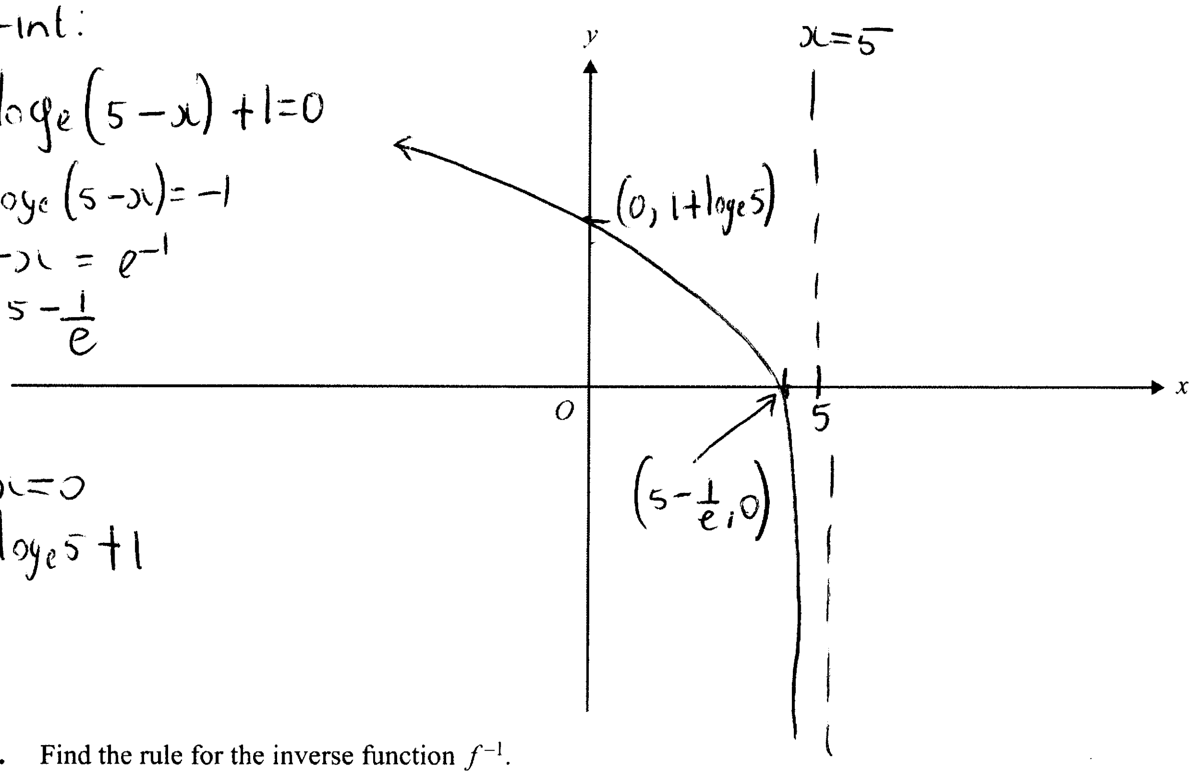
$5 - x = e^{-1}$

$x = 5 - \frac{1}{e}$

y-int:

Let $x = 0$

$y = \log_e 5 + 1$



b. Find the rule for the inverse function f^{-1}.

$x = \log_e(5 - y) + 1$

$x - 1 = \log_e(5 - y)$

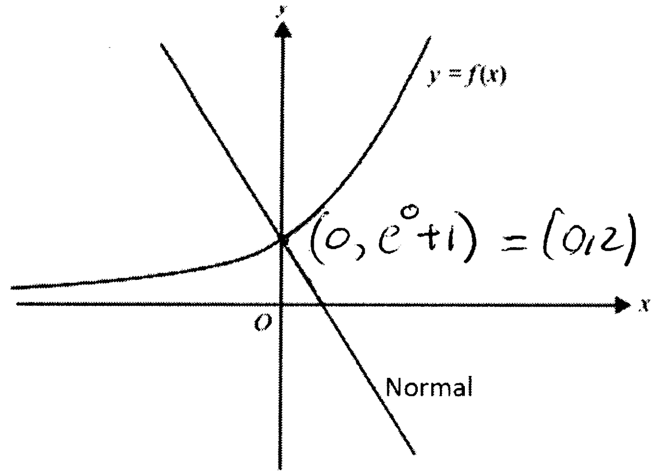
$5 - y = e^{x-1}$

$\therefore 5 - e^{x-1} = y$

$\therefore f^{-1}(x) = -e^{x-1} + 5$

3 + 2 = 5 marks

The graph of $f: R \to R, f(x) = e^{\frac{x}{2}} + 1$ is shown. The normal to the graph of f where it crosses the y- axis is also shown.



a. Find the equation of the **normal** to the graph of f where it crosses the y-axis.

$f'(x) = \frac{1}{2} e^{x/2} \quad \therefore f'(0) = \frac{1}{2} \times e^0 = \frac{1}{2}$

$\therefore$ Gradient of normal $= -2$.

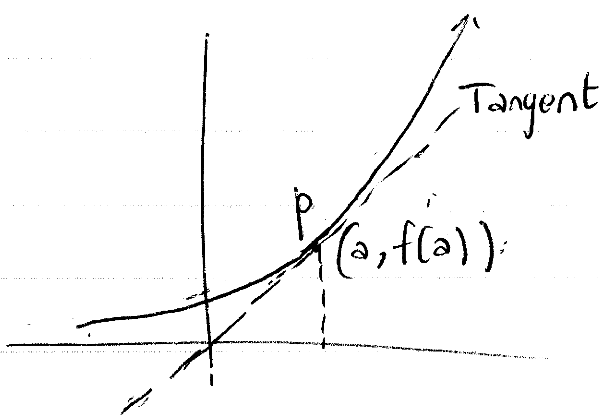
Normal goes through $(0,2)$

$\therefore$ Equation: $y = -2x + 2$

2 marks

Question 8

Let $f: R \to R, f(x) = e^x + k$, where k is a real number. The tangent to the graph of f at the point where $x = a$ passes through the point (0, 0). Find the value of k in terms of a.



Gradient of tangent at P $= f'(a)$

$f'(x) = e^x$

$\therefore f'(a) = e^a$

But gradient of tangent

$= \frac{f(a) - 0}{a - 0}$

$= \frac{e^a + k}{a}$

3 marks

$\therefore \frac{e^a + k}{a} = e^a$

$\therefore e^a + k = ae^a$

$\therefore k = ae^a - e^a$