SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

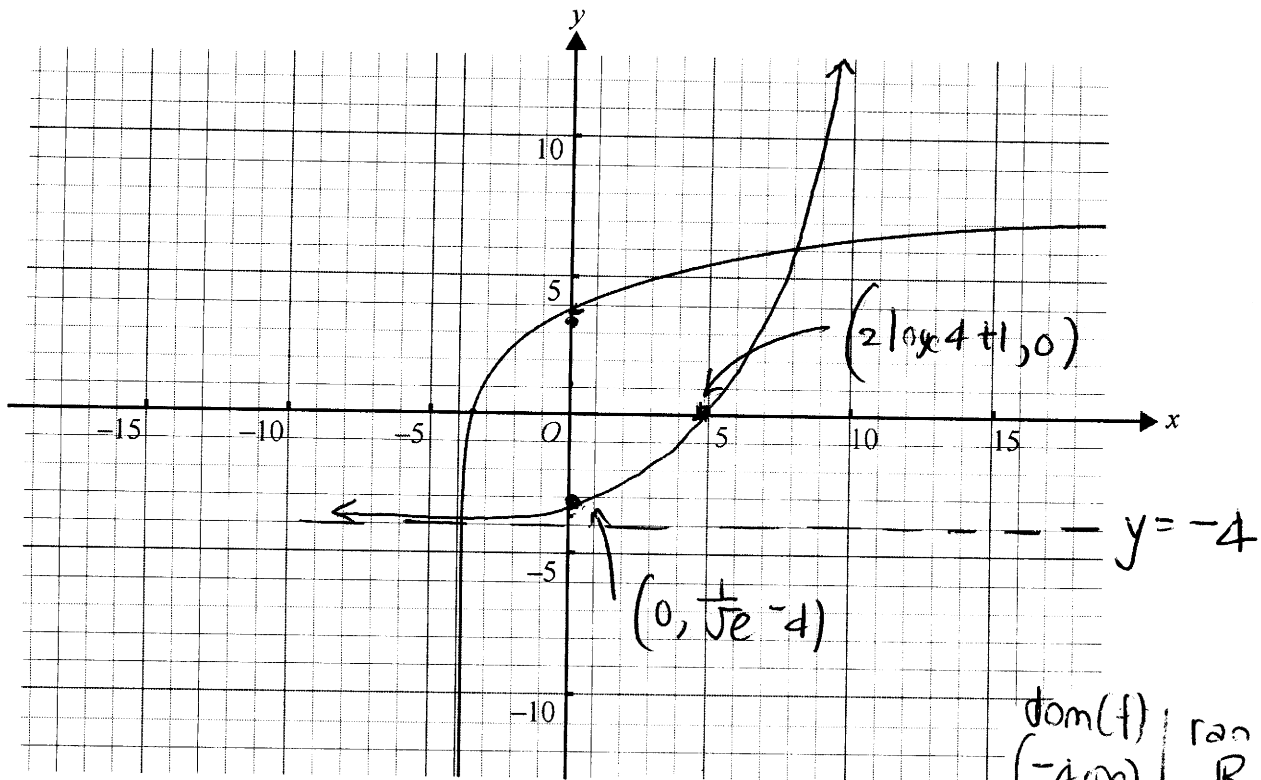
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Part of the graph of the function $g: (-4, \infty) \to R$, $g(x) = 2\log_e(x+4) + 1$ is shown on the axes below.



dom(f)	ran(f)
$(-4,\infty)$	R
dom(f^{-1})	ran(f^{-1})
R	$(-4,\infty)$

i. Find the rule and domain of g^{-1}, the inverse function of g.

$x = 2\log_e(y+4) + 1$

$\frac{x-1}{2} = \log_e(y+4)$

$y + 4 = e^{\frac{x-1}{2}}$ $\therefore f^{-1}: R \to R,\ f^{-1}(x) = e^{\frac{x-1}{2}} - 4$

ii. On the set of axes above sketch the graph of g^{-1}. Label the axes intercepts with their exact values.

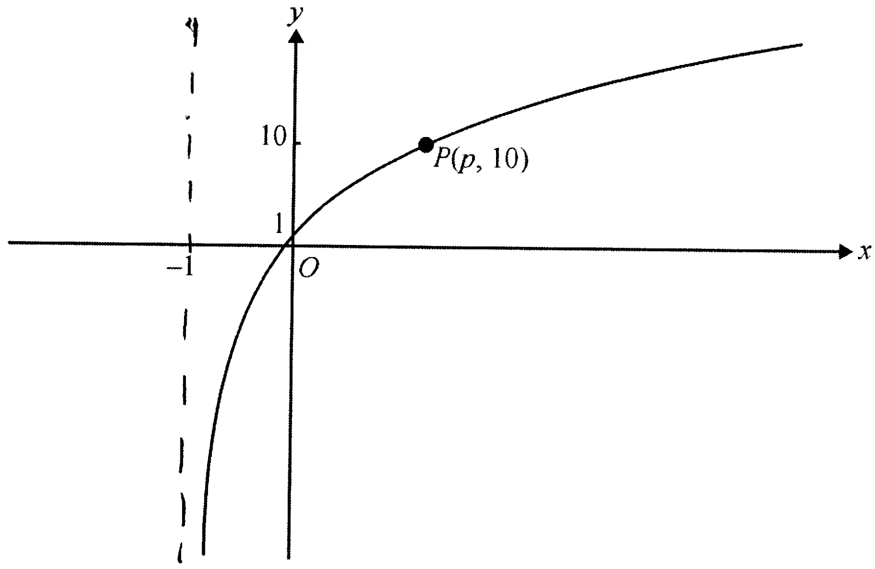
$y = 2\log_e(x+4) + 1$ $\therefore$ x-int of f^{-1} is $(2\log_e 4 + 1, 0)$

Let $x = 0$ $y = 2\log_e 4 + 1$ y-int: $(0, e^{-\frac{1}{2}} - 4)$

b. The diagram below shows part of the graph of the function with rule

$$f(x) = k\log_e(x + a) + c, \text{ where } k, a \text{ and } c \text{ are real constants.}$$

- The graph has a vertical asymptote with equation $x = -1$.
- The graph has a y-axis intercept at 1.
- The point P on the graph has coordinates $(p, 10)$, where p is another real constant.



i. State the value of a.

$a = 1$

ii. Find the value of c.

$f(0) = 1 \quad \therefore k\log_e(0+1) + c = 1$

$\therefore k\log_e 1 + c = 1$

$\therefore c = 1$

iii. Show that $k = \dfrac{9}{\log_e(p+1)}$.

$f(x) = k\log_e(x+1) + 1$

$f(p) = 10$

$\therefore 10 = k\log_e(p+1) + 1$

$\therefore 9 = k\log_e(p+1)$

$\therefore k = \dfrac{9}{\log_e(p+1)}$

iv. Show that the gradient of the tangent to the graph of f at the point P is $\dfrac{9}{(p+1)\log_e(p+1)}$.

$$f(x) = \frac{9}{\log_e(p+1)} \log_e(x+1) + 1$$

$$\therefore f'(x) = \frac{9}{(x+1)\log_e(p+1)}$$

$$\therefore f'(p) = \frac{9}{(p+1)\log_e(p+1)}$$

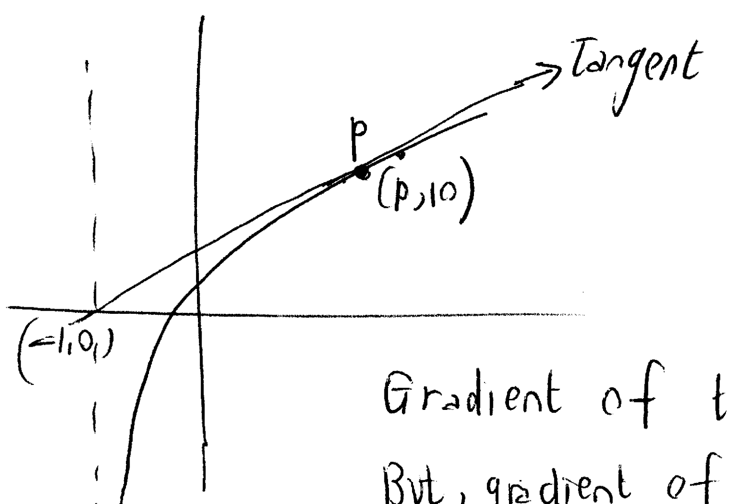
v. If the point $(-1, 0)$ lies on the tangent referred to in **part b.iv.**, find the exact value of p.

$$p = e^{\frac{9}{10}} - 1$$

(see below)

1 + 1 + 2 + 1 + 2 = 7 marks

Total 17 marks



Gradient of tangent $= f'(p)$

But, gradient of tangent $= \dfrac{10-0}{p--1}$

$$\therefore \frac{10}{p+1} = \frac{9}{(p+1)\log_e(p+1)}$$

$$\therefore \frac{10}{1} = \frac{9}{\log_e(p+1)}$$

$$\log_e(p+1) = \frac{9}{10}$$

$$p+1 = e^{\frac{9}{10}}$$

$$\therefore p = e^{\frac{9}{10}} - 1$$