

Quadratic graphs

Name: _____

- 1 For the graph of the equation $y = ax^2$ what is the effect of a value of:

- (a) $a = \frac{1}{4}$? Wider than $y = x^2$
(b) $a = -3$? Steeper and reflected

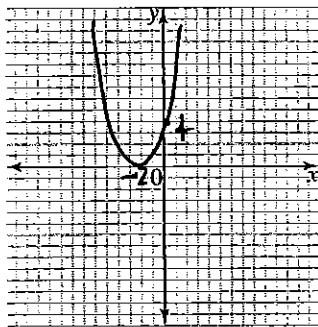
- 2 For the graph of the equation $y = x^2 + c$ what is the effect of a value of:

- (a) $c > 0$? Moved up the y -axis (above x -axis)
(b) $c < 0$? Moved down the y -axis (below x -axis)

- 3 For the graph of the equation $y = (x - b)^2$ what is the effect of a value of:

- (a) $b > 0$? Moved b units to right of origin
(b) $b < 0$? Moved b units to left of origin

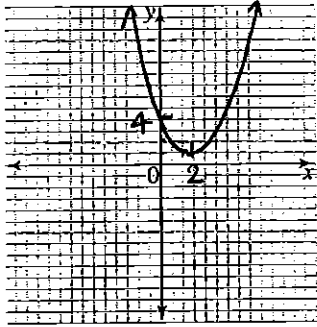
- 4 (a) Sketch the graph of $y = (x + 2)^2$ on the axes provided. (a)



- (b) Where is the turning point and the axis of symmetry?

T/p: $(-2, 0)$ Axis of symmetry: $x = -2$

- 5 (a) Sketch the graph of $y = (x - 2)^2$ on the axes provided.



- (b) Where is the turning point and the axis of symmetry?

T/p: $(2, 0)$ Axis of symmetry: $x = 2$

- 6 For the equation $y = -(x + 3)^2$ state:

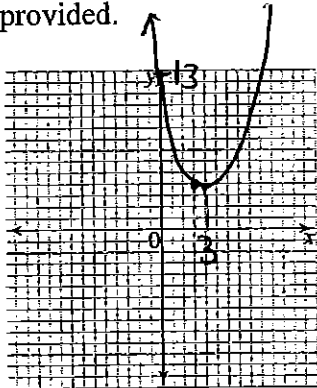
- (a) the co-ordinates of the turning point
 (b) if it is a maximum or minimum turning point
 (c) the equation of the axis of symmetry.

T/p: $(-3, 0)$

Maximum

$x = -3$

- 7 (a) Sketch the graph of $y = (x - 3)^2 + 4$ on the axes provided.



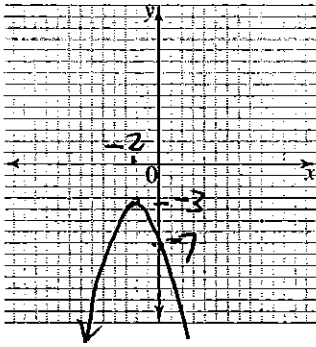
T/p: $(3, 4)$

y-int: $y = (0 - 3)^2 + 4$
 $= (-3)^2 + 4$
 $= 13$

- (b) Label the turning point and the axis of symmetry.

Axis of symmetry: $x = 3$

- 8 (a) Sketch the graph of $y = -(x + 2)^2 - 3$ on the axes provided.



$$T/p: (-2, -3)$$

- (b) Label the turning point, the y-intercept and the axis of symmetry.

Axis of symmetry: $x = -2$

y-int: let $x = 0$

$$y = -(0+2)^2 - 3 \\ = -7$$

- 9 For $y = (x + 1)^2 - 2$ state:

- (a) the co-ordinates of the turning point
(b) if it is a maximum or minimum turning point
(c) the equation of the axis of symmetry.

$$T/p: (-1, -2)$$

Minimum

$$x = -1$$

- 10 State a possible equation of a quadratic function with a minimum turning point of $(1, -4)$.

$$y = (x - 1)^2 - 4$$

Solving Quadratic Equations

1 Solve the quadratic equation

$$(2x - 7)^2 = 0.$$

$$(2x - 7)(2x - 7) = 0$$

\downarrow
 $x = \frac{7}{2}$ $x = \frac{7}{2}$

$$\therefore x = \frac{7}{2}$$

2 Solve the quadratic equation

$$(5x - 7)(-3x + 8) = 0.$$

$$(5x - 7)(8 - 3x) = 0$$

\downarrow \downarrow
 $x = \frac{7}{5}$ $x = \frac{8}{3}$

$$x = \frac{7}{5}, \frac{8}{3}$$

3 Solve the quadratic equation

$$7x^2 - 49x = 0.$$

$$7x(x - 7) = 0$$

$$x = 0, 7$$

4 Solve the quadratic equation

$$x^2 - 4x - 12 = 0.$$

Factors of -12 that add up to -4

$$(x - 6)(x + 2) = 0$$
$$x = 6, -2$$

5 Solve the quadratic equation

$$6(x - 13)^2 - 24 = 0.$$

$$6(x - 13)^2 = 24$$

$$(x - 13)^2 = 4$$

$$x - 13 = \pm\sqrt{4}$$

$$x - 13 = 2 \quad \text{or} \quad x - 13 = -2$$

$$x = 15, 11$$

6 Solve the quadratic equation

$$\frac{1}{6}x^2 - \frac{4}{7}x = 0.$$

$$x \left(\frac{x}{6} - \frac{4}{7} \right) = 0$$

$$x = 0 \quad \text{or} \quad \frac{x}{6} = \frac{4}{7}$$

$$\therefore x = 0, \frac{24}{7}$$

7 A ball is thrown vertically upwards. The height of the ball (h) after t seconds is given by the equation $h = 10t - 5t^2$. Calculate the time it takes for the ball to hit the ground.

$$h = 5t(2 - t)$$

$$h = 0 \text{ if } t = 0, 2$$

\therefore Takes 2 seconds

8 When a number is added to its square the result is 20. Find the number.

Let number = x

$$x^2 + x = 20$$

$$x^2 + x - 20 = 0$$

$$(x + 5)(x - 4) = 0$$

$$x = -5, 4.$$

- 9 When 5 is subtracted from twice the square of a whole number the result is 9 times the original number. What is the whole number?

$$2x^2 - 5 = 9x$$

$$2x^2 - 9x - 5 = 0$$

$$2x^2 - 10x + x - 5 = 0$$

$$2x(x-5) + (x-5) = 0$$

$$(x-5)(2x+1) = 0$$

$$x = 5, -\frac{1}{2}$$

Factors of -10 that add to -9 are -10, 1

- 10 The distance (d) travelled by a bicycle which is accelerating is given by the formula $d = t^2 + 20t$ where t is measured in minutes. If the distance travelled was 300 metres, for how long was it travelling?

$$d = t^2 + 20t$$

$$300 = t^2 + 20t$$

$$t^2 + 20t - 300 = 0$$

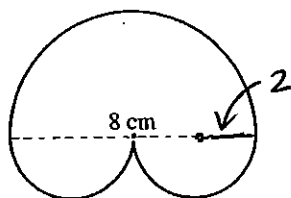
$$(t+30)(t-10) = 0$$

$$t = -30, 10$$

since $t \geq 0$,
 $t = 10$.

Measurement

- 11 Determine the perimeter of the object shown in the figure below. Give answer correct to 2 decimal places.



$$P = \text{Big semi} + 2 \text{ small semi}$$

$$= \text{Big semi} + 1 \text{ small circle}$$

$$= \frac{1}{2} \times 2\pi \times (4) + 2\pi \times (2)$$

$$= 4\pi + 4\pi$$

$$= 8\pi$$

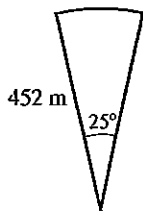
$$= 8 \times 3.141 \dots$$

$$= 25.13 \text{ cm}$$

- 12 Determine the area of the object in question 1.
Give answer correct to 2 decimal places.

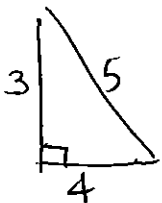
$$\begin{aligned}
 A &= \frac{\pi \times R^2}{2} + \pi r^2 \\
 &= \frac{\pi \times 4^2}{2} + \pi \times 2^2 \\
 &= 8\pi + 4\pi \\
 &= 12\pi = 37.7 \text{ cm}
 \end{aligned}$$

- 13 Find the area of the sector of a circle shown in the figure below. The radius of the circle is 452 m. Give answer correct to 2 decimal places.

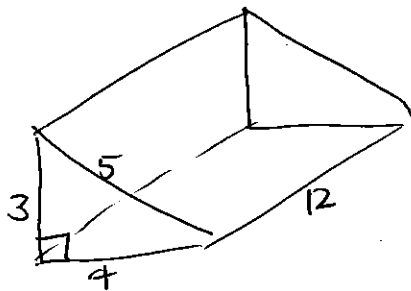


$$\begin{aligned}
 A &= \frac{\theta}{360} \times \pi r^2 \\
 &= \frac{25}{360} \times \pi \times 452^2 \\
 &= 44572.22 \text{ m}^2
 \end{aligned}$$

- 14 A prism has as its base a right-angled triangle of sides 3, 4 and 5 cm respectively. The prism is 12 cm long. Determine its surface area.

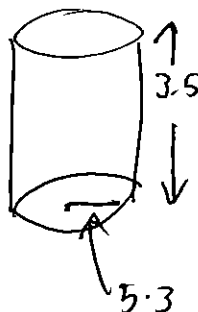


$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 4 \times 3 \\
 &= 6 \text{ cm}^2
 \end{aligned}$$



$$\begin{aligned}
 \text{T.S.A.} &= 2 \times A_{\text{triangle}} \\
 &\quad + A_{\text{base}} + A_{\text{front}} \\
 &\quad + A_{\text{back}} \\
 &= 2 \times 6 + 4 \times 12 + 5 \times 12 \\
 &\quad + 3 \times 12 \\
 &= 12 + 48 + 60 + 36 \\
 &= 156 \text{ cm}^2
 \end{aligned}$$

- 15 Determine the surface area of a cylinder whose radius is 5.3 cm and height is 3.5 cm. Give answer correct to 2 decimal places.



$$\begin{aligned}
 \text{T.S.A.} &= 2\pi rh + 2\pi r^2 \\
 &= 2\pi \times 5.3 \times 3.5 + 2\pi \times 5.3^2 \\
 &= 293.05 \text{ cm}^2
 \end{aligned}$$

- 16 Determine the volume of the cylinder in question 5. Give answer correct to 2 decimal places.

$$V = \pi r^2 h = \pi \times 5.3^2 \times 3.5$$

$$= 308.87 \text{ cm}^3$$

- 17 A sphere has a radius of 4.5 cm. Find:
(a) the volume

$$V = \frac{4\pi r^3}{3} \quad \therefore V = \frac{4}{3} \times \pi \times (4.5)^3 = 381.7 \text{ cm}^3$$

- (b) the surface area.

$$S = 4\pi r^2$$

Give answers correct to 2 decimal places.

$$= 4\pi \times (4.5)^2$$

$$= 254.47 \text{ cm}^2$$

- 18 Convert the following measures of volume to the units specified.

(a) 5.6 litres = 5600 cm^3

(b) 3.1 m^3 = ~~31000~~ 3100 litres

(c) 0.0045 m^3 = 4500 cm^3

$$1000 \text{ ml} = 1 \text{ L}$$

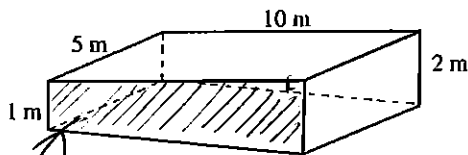
$$1 \text{ m}^3 = 1000000 \text{ cm}^3$$

$$\therefore 5.6 \text{ L} = 5.6 \times 1000 \text{ cm}^3$$

$$= 5600 \text{ cm}^3$$

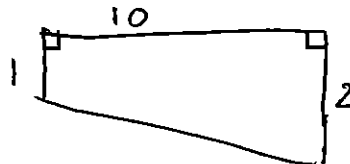
$$1 \text{ m}^3 = 1000 \text{ L}$$

- 19 The figure below shows a home swimming pool.



Find the capacity of the pool in litres.

Cross-section



$$A = \frac{1}{2} h (a+b)$$

$$= \frac{1}{2} \times 2 \times (1+10)$$

$$= 11 \text{ m}^2$$

$$\rightarrow 0.0045 \text{ m}^3$$

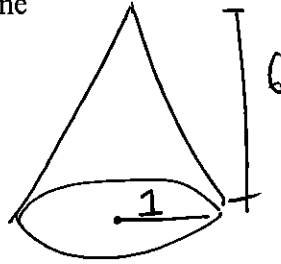
$$= 0.0045 \times 1000 \times 1000 \text{ cm}^3$$

$$\therefore V = 11 \times 2$$

$$= 22 \text{ m}^3$$

- 20 A monument in the shape of a cone has a base diameter of 2 m and a height of 6 m. Determine the volume of the cone correct to 2 decimal places.

$$V = \frac{\pi r^2 h}{3}$$



$$= \frac{\pi \times 1^2 \times 6}{3} = 2\pi = 6.28 \text{ m}^3$$

Probability

- 21 In the Tattsлото draw there are 45 numbered balls. Find the probability that the first number drawn is:

(a) a 1

$$\frac{1}{45}$$

(b) a 45

$$\frac{1}{45}$$

(c) even

$$\frac{22}{45}$$

(d) odd

$$\frac{23}{45}$$

(e) greater than 40.

Nos greater than 40: $\{41, 42, 43, 44, 45\} \therefore \frac{5}{45} = \frac{1}{9}$

- 22 A bag contains 3 white balls, 4 red balls, 5 black balls, 6 green balls and 7 yellow balls. A single ball is drawn from the bag. What is the probability that it is neither white or black?

$$\text{TOTAL} = 3 + 4 + 5 + 6 + 7 = 25$$

$$\text{No. of white \& black} = 3 + 5 = 8.$$

$$\therefore \text{Pr (neither white nor black)} = \frac{17}{25}$$

- 23 Consider the following table, which shows the number of cars repaired by 5 different mechanics during a day.

| | | | | | |
|------------|---|----|----|----|----|
| Mechanic | 1 | 2 | 3 | 4 | 5 |
| Cars fixed | 8 | 15 | 12 | 22 | 16 |

A customer returns his car because it was not repaired properly. Without knowing which mechanic worked on it, determine the probability that it was mechanic 1.

$$\frac{8}{(8+15+12+22+16)} = \frac{8}{73}$$

- 24 In a survey of voters in Victoria, 43% were in favour of increasing the size of the army.

If the survey asked 6500 people, how many were *not* in favour of increasing the size?

$$0.57 \times 6500 = 3,705$$

- 25 Consider the following table showing voter preferences in all 6 Australian states.

| | Labor | Liberal | Democrat | |
|-----|-------|---------|----------|--------|
| QLD | 340 | 620 | 190 | 1150 |
| NSW | 560 | 442 | 219 | 1221 |
| VIC | 618 | 589 | 218 | 1426 |
| TAS | 307 | 419 | 167 | 893 |
| SA | 478 | 462 | 226 | 1166 |
| WA | 712 | 423 | 178 | 1313 |
| | | | | + 1198 |

$$\begin{aligned} \text{Total no. of voters} &= 1150 + 1221 + 1426 + 893 \\ &\quad + 1166 + 1313 \\ &= 7169 \end{aligned}$$

Determine the relative frequency of Democrat supporters.

$$\begin{aligned} \text{Total no. of Democrat voters} &= 1198 \end{aligned}$$

$$\begin{aligned} \therefore \text{Relative frequency} &= \frac{1198}{7169} \\ &= 16.71\% \end{aligned}$$

26 Using the table in question 5, if a voter who participated in the survey is chosen at random, find the probability (as a decimal correct to two decimal places) that:

(a) they supported the Labor party $\frac{3015}{7169} \approx 0.42$

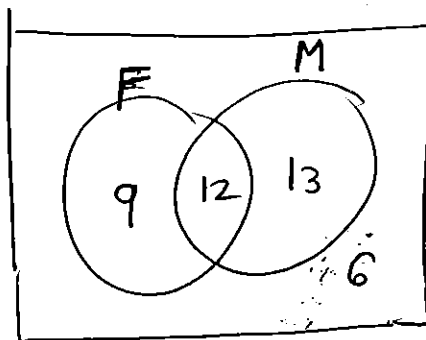
(b) they supported the Liberal party

$$\frac{2956}{7169} = 0.41$$

(c) they are a Victorian Democrat supporter.

$$\frac{218}{7169} = 0.03$$

27 In a class of 40 students, 12 liked both fish and meat, 6 liked neither. If there were a total of 25 who liked meat, construct a Venn Diagram of this situation.



$$n(\mathcal{E}) = 40$$

28 In a gambling game, you win if you draw a 'winning card' from a normal deck of 52 playing cards. Winning cards are:

- any spade
- king of hearts
- queen of diamonds
- jack of clubs
- any diamond in the range 2 to 10.

Is this a 'fair' game?

No. of winning cards

$$= 13 + 1 + 1 + 1 + 9$$

$$= 25 \text{ cards}$$

$$\text{Pr}(\text{winning}) = \frac{25}{52}$$

$$\text{Pr}(\text{losing}) = \frac{27}{52}$$

No, not fair
 "fair" means equal chance of winning as losing.

29 In a marathon, the probability that Mikayla Jones will win is quoted as $\frac{1}{9}$. Determine the correct odds in this situation.

$$\begin{array}{l} \text{lose} : \text{win} \\ \frac{8}{9} : \frac{1}{9} \\ \text{odds} : 8-1 \end{array}$$

30 Rhonda had a \$40 bet on Brew in the Melbourne Cup. If Brew's odds were 15-1 calculate the amount of Rhonda's payout.

$$\begin{array}{c} 15-1 \\ \swarrow \quad \searrow \\ \times 40 \quad \times 40 \\ \downarrow \quad \downarrow \\ 600 : 40 \end{array}$$

$$\therefore \text{Wins } \$600$$

$$\text{payout} = \$640$$