



SOLUTIONS

Y10 MM - 2014

Number and algebra: Real numbers Unit Test

Name: _____

Part A: Vocabulary	4 marks
Part B: Multiple Choice questions	10 marks
Part C: Short answer questions	14 marks
Part D: Analysis questions	12 marks
Total	40 marks

Part A: Vocab

Please choose the right missing word from the table below:

- 1- A rational number is one that can be expressed as a fraction.
- 2- All fractions can be written as finite or recurring decimals.
- 3- Only like Surds can be added or subtracted.

WORD LIST

Real Numbers	like	surds
finite	rational	Irrational
non-recurring	recurring	

Part B: Multiple Choice Questions

1 Which of the following numbers is irrational?

- A $\sqrt{25}$
- B $\sqrt{30}$
- C $\sqrt{36}$
- D $\sqrt{49}$
- E $\sqrt{\frac{9}{4}}$

1

2. Consider the expressions: $\sqrt{3m}$, $\sqrt{64m}$, $\sqrt{\frac{m}{81}}$, $\sqrt[3]{3m}$, $\sqrt{\frac{45}{m}}$. If $m = 9$, how many of these expressions will evaluate to surds?

- A. None
- B. One
- C. Two
- D. Three
- E. Four

$$\begin{aligned} \sqrt{27} &= 3\sqrt{3} \text{ surd} & \sqrt[3]{27} &= 3 \\ \sqrt{64 \times 9} &= 8 \times 3 & \sqrt{\frac{45}{9}} &= \sqrt{5} \text{ surd} \\ \sqrt{\frac{9}{81}} &= \sqrt{\frac{1}{9}} = \frac{1}{3} \end{aligned}$$

1

3 When fully simplified, $5\sqrt{5} \times 3\sqrt{10}$ is equal to:

- A $75\sqrt{2}$
- B $15\sqrt{5}$
- C $150\sqrt{5}$
- D $15\sqrt{50}$
- E $8\sqrt{50}$

$$\begin{aligned} 5\sqrt{5} \times 3\sqrt{10} \\ &= 15\sqrt{50} \\ &= 15\sqrt{25} \sqrt{2} = 75\sqrt{2} \end{aligned}$$

1

4 What is $\frac{5}{3\sqrt{2}}$ fully simplified with a rational denominator equal to?

- A $\frac{15\sqrt{2}}{18}$
- B $\frac{5\sqrt{2}}{2}$
- C $\frac{5\sqrt{2}}{3}$
- D $\frac{5\sqrt{2}}{6}$
- E $\frac{\sqrt{10}}{6}$

$$\begin{aligned} \frac{5}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{5\sqrt{2}}{3 \times 2} \\ &= \frac{5\sqrt{2}}{6} \end{aligned}$$

1

<p>5 When expressed in its simplest form, $\sqrt{36x^5}$ is equal to:</p> <p>A $x^2\sqrt{6x}$ <input checked="" type="radio"/> B $6x^2\sqrt{x}$ C $x^4\sqrt{6x}$ D $6x^4\sqrt{x}$ E $x^2\sqrt{36x}$</p>	1
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$$\begin{aligned} &\sqrt{36x^5} \\ &= \sqrt{36} \sqrt{x^4} \sqrt{x} \\ &= 6x^2\sqrt{x} \end{aligned}$$

<p>6 The value of $\left(\frac{5}{8}\right)^{-1}$ is:</p> <p>A $-1\frac{3}{5}$ B $-\frac{5}{8}$ C $\frac{5}{8}$ <input checked="" type="radio"/> D $1\frac{3}{5}$ E -40</p>	1
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$$\left(\frac{5}{8}\right)^{-1} = \frac{8}{5} = 1\frac{3}{5}$$

<p>7 The value for x in $2^{x-1} = 30$ to 3 significant figures is:</p> <p>A 3.91 <input checked="" type="radio"/> B 5.91 C 3.907 D 5.907 E 4.907</p>	1
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Solve $(2^{x-1} = 30, x)$
 $x = 5.91$

<p>8 If the equation $y = a^x$ is expressed in logarithmic form, it will become:</p> <p>A. $x = \log_y a$ <input checked="" type="radio"/> B. $x = \log_a y$ C. $y = \log_x a$ D. $y = \log_a x$ E. $a = \log_y x$</p>	1
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$$\begin{aligned} y &= a^x \\ \therefore x &= \log_a y \end{aligned}$$

<p>9 When simplified, $\sqrt{x} \times \sqrt{16x^5}$ becomes:</p> <p>A $8x^6$</p> <p>B $8x^{\frac{5}{2}}$</p> <p>C $4x^6$</p> <p><input checked="" type="radio"/> D $4x^3$</p> <p>E $4x^{\frac{5}{2}}$</p>	1
<p>10 What is the conjugate surd of $6\sqrt{5} - 13$?</p> <p><input checked="" type="radio"/> A $6\sqrt{5} + 13$</p> <p>B $6\sqrt{5} - 13$</p> <p>C $13 - 6\sqrt{5}$</p> <p>D $\frac{1}{6\sqrt{5}+13}$</p> <p>E $\frac{1}{6\sqrt{5}-13}$</p>	1

$$\begin{aligned}
 &\sqrt{x} \times \sqrt{16x^5} \\
 &= \sqrt{16} \sqrt{x} \sqrt{x^5} \\
 &= 4 \sqrt{x^6} \\
 &= 4x^3
 \end{aligned}$$

Part C: Short answer questions

Question 1

Simplify each of the following. To achieve full marks all steps need to be shown.

(a) $\sqrt[4]{81a^8b^{24}}$

$$\begin{aligned} & \sqrt[4]{81a^8b^{24}} \\ &= (81a^8b^{24})^{\frac{1}{4}} \\ &= 81^{\frac{1}{4}} a^{\frac{8}{4}} b^{\frac{24}{4}} = 3a^2b^6 \end{aligned}$$

2 marks

(b) $\sqrt{80} + 5\sqrt{20} - 6\sqrt{125}$

$$\begin{aligned} & \sqrt{80} + 5\sqrt{20} - 6\sqrt{125} \\ &= \sqrt{16 \times 5} + 5\sqrt{4 \times 5} - 6\sqrt{25 \times 5} \\ &= 4\sqrt{5} + 10\sqrt{5} - 30\sqrt{5} \\ &= -16\sqrt{5} \end{aligned}$$

2 marks

(c) $\frac{5\sqrt{42} \times 4\sqrt{6}}{10\sqrt{7}}$

$$\begin{aligned} &= \frac{20\sqrt{42}\sqrt{6}}{10\sqrt{7}} \\ &= 2 \times \sqrt{\frac{42}{7}} \times \sqrt{6} \\ &= 2 \times \sqrt{6} \times \sqrt{6} \\ &= 2 \times 6 \\ &= 12 \end{aligned}$$

2 marks

Question 2

Rationalize the denominator for the following:

$$(a) \frac{2\sqrt{11} + 7\sqrt{15}}{\sqrt{3}}$$

$$= \frac{2\sqrt{11}}{\sqrt{3}} + 7 \frac{\sqrt{15}}{\sqrt{3}} = \frac{2\sqrt{33}}{3} + 7\sqrt{5}$$

$$= \frac{2\sqrt{11}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + 7 \sqrt{\frac{15}{3}} \therefore \frac{2\sqrt{33}}{3} + 7\sqrt{5}$$

$$(b) \frac{16}{\sqrt{13}-3}$$

$$\frac{6}{\sqrt{13}-3} \times \frac{\sqrt{13}+3}{\sqrt{13}+3} = \frac{6(\sqrt{13}+3)}{13-9}$$

$$= \frac{6(\sqrt{13}+3)}{4} = \frac{3(\sqrt{13}+3)}{2}$$

(2 + 3 = 5 marks)

Question 3

Write the following equation in logarithm form:

$$\sqrt{81} = 9$$

$$81^{\frac{1}{2}} = 9$$

$$\therefore \frac{1}{2} = \log_{81} 9$$

1 mark

Question 4The Richter scale is used to describe the energy in earthquakes. The R value for an earthquake with K units of energy is:

$$R = \frac{2}{3} \log_{10} K - 0.9$$

Without using CAS, use this formula to calculate the R value for an earthquake with 1,000,000 units of energy. Show all your working.

$$\log_{10} 1000000 = 6 \quad \text{since} \quad 10^6 = 1,000,000$$

$$\therefore R = \frac{2}{3} \log_{10} (1000000) - 0.9$$

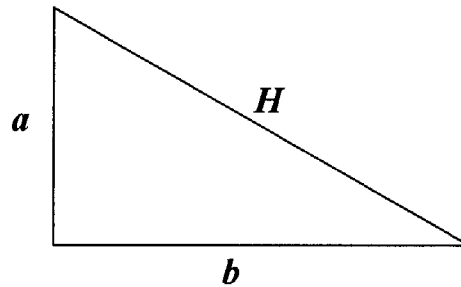
$$= \frac{2}{3} \times 6 - 0.9 = 4 - 0.9 = 3.1$$

Part D: Analysis Questions

2 marks

All working must be shown to gain full marks.

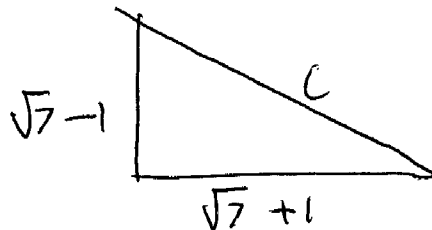
Question 1



In the right-angled triangle above:

$$a = \sqrt{7} - 1 \text{ and } b = \sqrt{7} + 1$$

- a. Use Pythagoras' Theorem to find the value of H . Show all working.



$$C^2 = (\sqrt{7} - 1)^2 + (\sqrt{7} + 1)^2$$

$$\therefore C^2 = 7 - 2\sqrt{7} + 1 + 7 + 2\sqrt{7} + 1$$
$$= 16$$

$$\therefore C = \sqrt{16} = 4 \quad \therefore H = 4$$

4 marks

- b. Find the area of the triangle above.

$$(\text{Area of triangle} = \frac{1}{2}bh)$$

$$A = \frac{1}{2}bh$$

$$\therefore A = \frac{1}{2} \times (\sqrt{7} - 1) \times (\sqrt{7} + 1)$$

$$= \frac{1}{2} \times (7 + \sqrt{7} - \sqrt{7} - 1)$$

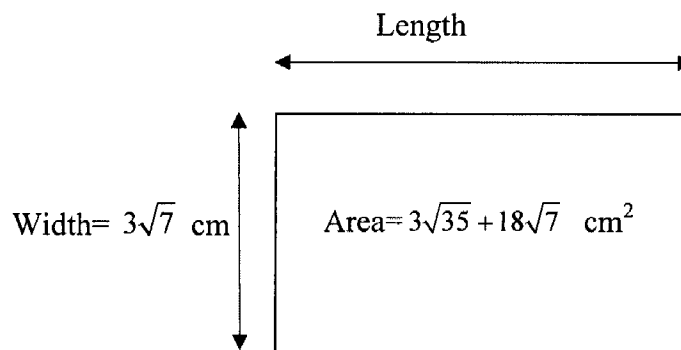
$$= \frac{1}{2} \times 6 = 3$$

3 marks

Question 2

A rectangle has an area of $3\sqrt{35} + 18\sqrt{7}$ square centimetres, and a width of $3\sqrt{7}$ cm.

Find the exact value of the length of the rectangle.



$$A = lw$$

$$\therefore 3\sqrt{35} + 18\sqrt{7} = 3\sqrt{7} \times l$$

$$\therefore \frac{3\sqrt{35} + 18\sqrt{7}}{3\sqrt{7}} = l$$

$$\therefore l = \frac{3\sqrt{35}}{3\sqrt{7}} + \frac{18\sqrt{7}}{3\sqrt{7}}$$

$$= \frac{\sqrt{35}}{\sqrt{7}} + 6$$

$$= \sqrt{\frac{35}{7}} + 6$$

$$= \sqrt{5} + 6$$

5 marks