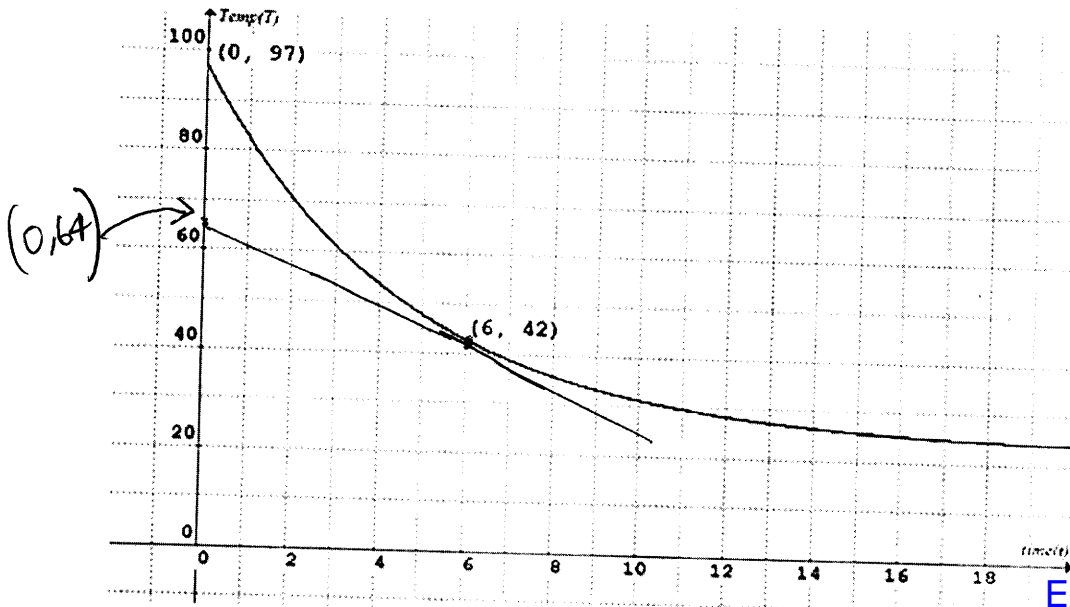


Question 1

Name: \_\_\_\_\_

The graph shown represents the temperature ( $T^\circ$  Celsius) of a cup of coffee as it cools over a period of time ( $t$  minutes).



- a Find the average rate of cooling over the first 6 minutes.

$$\frac{97 - 42}{0 - 6} = \frac{-55}{6} = -9\frac{1}{6} \text{ } ^\circ\text{C/min}$$

Exact value required if no mention of no. of decimal places.

- b By drawing a tangent to the curve, estimate the instantaneous rate of cooling at  $t = 6$

$$\frac{64 - 42}{0 - 6} = \frac{-22}{6} = -3\frac{2}{3} \text{ } ^\circ\text{C/min} \quad 2+2=4 \text{ marks}$$

$$\approx -4 \text{ } ^\circ\text{C/min}$$

Draw a tangent to the graph at  $t = 6$ , estimate the co-ordinates of a point on it,

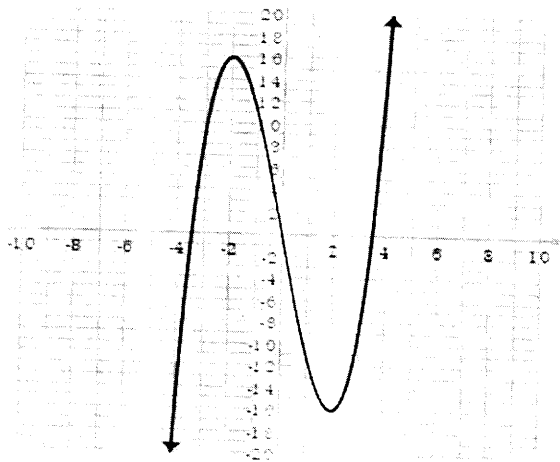
and then use the gradient formula to estimate the gradient of the tangent

Question 2 Consider the function shown to the right:

State the values of  $x$  for which this is an

increasing function.

$$x \in (-\infty, -2) \cup (2, \infty)$$



1 mark

**Question 3**

Consider the function  $g(x) = \sqrt{x-2}$

- a. Determine the average rate of change of the function  $g(x)$  over the interval  $x \in [6,11]$

$$\frac{g(11) - g(6)}{11 - 6} = \frac{\sqrt{9} - \sqrt{4}}{5} = \frac{1}{5}$$

- b. Estimate the instantaneous rate of change of the function  $g(x)$  at the point  $P$  where  $x = 3$  by considering points closer and closer to point  $P$  on both the right and left hand side.

| $x$   | $m_{\text{secant}}$                                |
|-------|--|
| 3.01  | $\frac{g(3.01) - g(3)}{3.01 - 3} \approx 0.498756$ |
| 2.99  | $\frac{g(2.99) - g(3)}{2.99 - 3} \approx 0.501256$ |
| 3.001 | $\frac{g(3.001) - g(3)}{0.001} \approx 0.4998751$  |
| 2.999 | $\frac{g(2.999) - g(3)}{0.001} \approx 0.50012506$ |

To see the limit, five or six decimal places should have been used.

∴ Estimate for instantaneous rate of change is 0.5.

2 + 3 = 5 marks

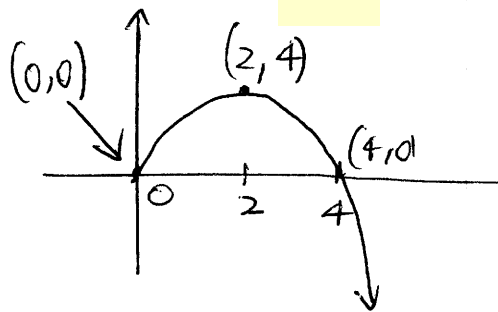
**Question 4**

A particle is moving along the  $x$  axis such that its position at time  $t$  seconds relative to the origin is:

$$x(t) = 4t - t^2$$

**NOTE domain!**

- a. Sketch the graph of  $x$  versus  $t$  for  $t \geq 0$ . Show the co-ordinates of all key points.



- b. At what value of  $t$  does the particle change its direction of motion?

$t = 2$  At the turning point, the particle instantaneously comes to rest and then begins to move in the opposite direction.

- c. For what values of  $t$  is the particle moving to the left?

$t > 2$  The particle is moving to the left when the gradient of the tangents to the curve are negative.

- d. Calculate the average velocity of the particle over the interval  $t \in [1,3]$

$$\frac{x(3) - x(1)}{3 - 1} = \frac{3 - 3}{2} = 0$$

2 + 1 + 1 + 1 = 5 marks