SOLUTIONS

Q1(a)

x-intercept: let $y=0$ $\quad 0 = \frac{45}{x+20} + 30$

$$-30 = \frac{45}{x+20}$$

$$-2 = \frac{3}{x+20}$$

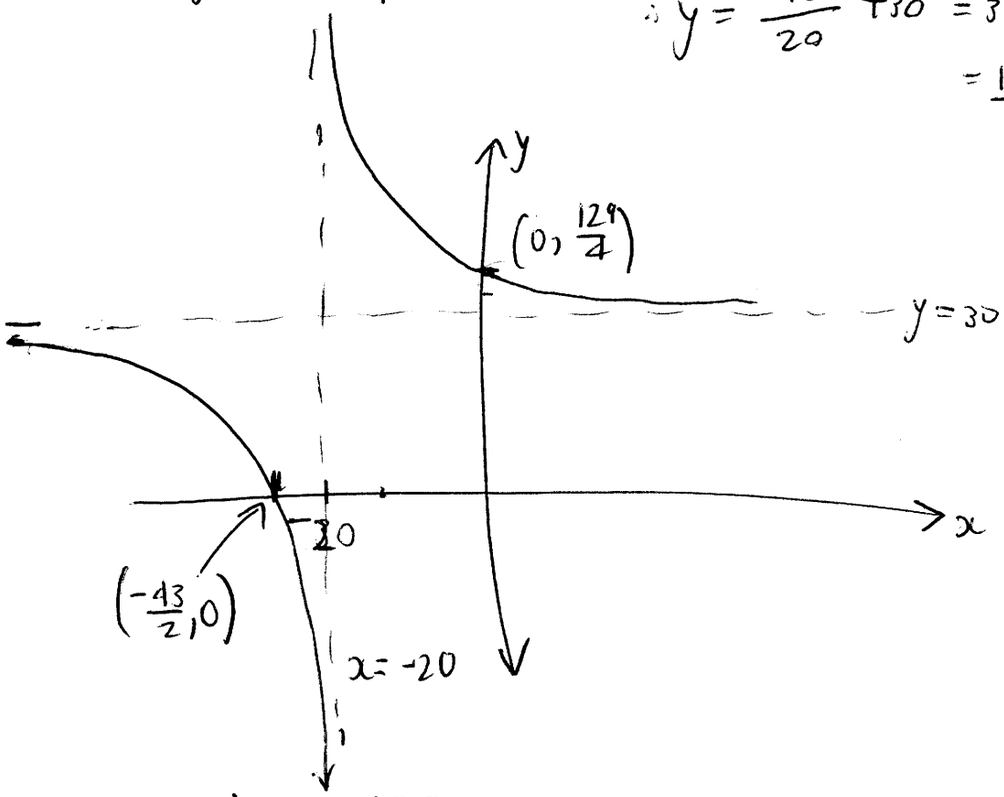
$$-2x - 40 = 3$$

$$x = -\frac{43}{2} \quad \left(-\frac{43}{2}, 0\right)$$

y-intercept: let $x=0$

$$\therefore y = \frac{45}{20} + 30 = 30 + \frac{9}{4}$$

$$= \frac{129}{4} \quad \therefore \left(0, \frac{129}{4}\right)$$



(b) $\frac{129}{4}$ metres

(c) $\frac{43}{2}$ metres

(d) $\sqrt{(80-0)^2 + (30.45 - 0)^2}$

≈ 85.60

(e) $h = 20,\ k = 38$

(f) $F(x) = \frac{-100}{x+20} + 38$

Let $x = 0 \quad F(0) = \frac{-100}{20} + 38 = 33$

$\therefore M = (0, 33)$.

(g)

$$f(x) = \frac{a}{x-b} + c$$

$\text{dom}(f)$	$\text{ran}(f)$
$\mathbb{R}\setminus\{b\}$	$\mathbb{R}\setminus\{c\}$
$\text{dom}(f^{-1})$	$\text{ran}(f^{-1})$
$\mathbb{R}\setminus\{c\}$	$\mathbb{R}\setminus\{b\}$

$$y = \frac{a}{x-b} + c$$

$\downarrow$

$$x = \frac{a}{y-b} + c$$

$$x - c = \frac{a}{y-b}$$

$$y - b = \frac{a}{x-c}$$

$$f^{-1}(x) = \frac{a}{x-c} + b, \quad x \in \mathbb{R}\setminus\{c\}$$

(h) If the two graphs intersect, they will intersect on the line $y = x$

$$y = x \quad (1)$$

$$y = -\frac{2}{x-3} + 5 \quad (2)$$

$$-\frac{2}{x-3} + 5 = x$$

$$\therefore \frac{-2}{x-3} = x - 5$$

$$-2 = (x-5)(x-3)$$

$$x^2 - 8x + 15 = -2$$

$$x^2 - 8x + 17 = 0$$

$$\Delta = (-8)^2 - 4 \times 17 = -4$$

No solution, since $\Delta < 0$.

(b) [cont] Therefore, f and f^{-1} do not intersect, and this will be a suitable flight path.

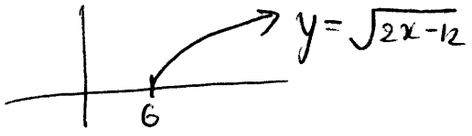
Q2. (a) $2x - 12 \geq 0$

$x \geq 6$

$\therefore a = 6$

(b) $g(f(x))$ is defined if $\text{ran}(f) \subseteq \text{dom}(g)$

$\text{ran}(f) = [0, \infty)$



$\text{dom}(g) = [0, \infty)$

$\therefore \text{ran}(f) \subseteq \text{dom}(g)$ is satisfied.

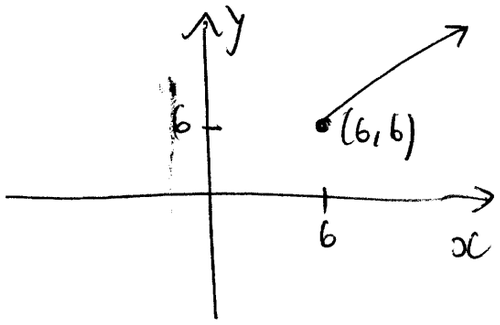
(c) $g(f(x)) = \frac{1}{2}\left(\sqrt{2x-12}\right)^2 + 6 \quad , x \geq 6$

$= \frac{1}{2}(2x-12) + 6 \quad , x \geq 6$

$= x - 6 + 6, \quad x \geq 6$

$= x, \; x \geq 6.$

dom(g)	ran(g)
$[0, \infty)$	$[6, \infty)$
dom(f)	**ran(f)**
$[6, \infty)$	$[0, \infty)$



$g(f(x)) = x, \quad x \in [6, \infty)$

(d) $f(g(x)) = \sqrt{2\left(\frac{x^2}{2}+6\right)-12}$, $x \geq 0$

$= \sqrt{x^2+12-12}$, $x \geq 0$

$= \sqrt{x^2}$, $x \geq 0$

$= |x|$, $x \geq 0$

$= x$, $x \geq 0$.

[NOTE: $\text{dom}(f(g(x))) = \text{dom}(g(x)) = [0, \infty)$]

(e) f and g are inverse functions.

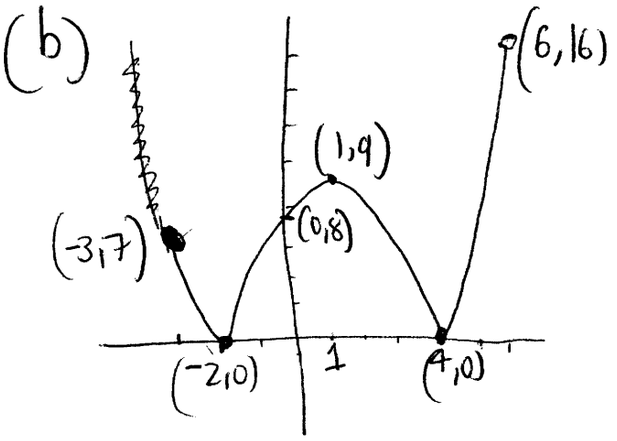
Q3.

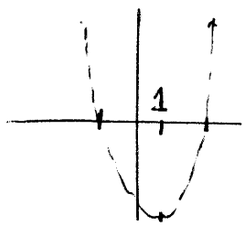
(a) $g(x) = x^2 - 2x + 1^2 - 1^2 - 8$

$= (x-1)^2 - 9$

T/p: $(1, -9)$

(b)





$y = (x-1)^2 - 9$

T/p: $(1, -9)$

x-int: $(-2, 0)$, $(4, 0)$

y-int: $(0, -8)$

If $x = -3$, $y = 7$

If $x = 6$, $y = 16$.

(c)