ANSWERS

Question 1

A virus is invading the cells of an organism. The number of infected cells N at time t days is:

$N = 25 \times 1.12^t$

a. How many cells are initially infected?

b. Find the number of cells infected after 3 days.

c. The organism will die if the number of infected cells exceeds 1500. How long will it take for this to occur? Give your answer correct to one decimal place.

Question 2

The amount D of a radioactive element present after t years is modelled by the rule:

$D = D_0 \times 2^{-kt}$ where $D_0 = 10$ kg.

a. What is the initial amount of the element present?

Let $t=0$ $\quad D = 10 \times 2^0 = 10 \times 1 = 10$ $\Rightarrow$ Initial amount $= 10kg$

b. If the amount of the element present is after 20 years is equal to 5 kg, find the value of k.

$D = 10 \times 2^{-kt}$

When $t = 20$, $D=5$ $\quad \therefore 5 = 10 \times 2^{-20k}$ $\Rightarrow$ $\therefore \frac{1}{2} = 2^{-20k}$ $\quad \therefore 2^{-1} = 2^{-20k}$ $\Rightarrow$ $-1 = -20k$ $\quad k = \frac{1}{20} = 0.05$

c. Determine after how many years the amount of the element present falls to 2 kg.

$D = 10 \times 2^{-\frac{t}{20}}$

Let $\quad 2 = 10 \times 2^{-\frac{t}{20}}$

$\therefore t \simeq 46.4$ years

Question 3

The value V of a car after x years is modelled by the rule: $V = 20000 \times 10^{-0.03x}$

a. What is the initial value of the car?

When $t=0$, $V = 20000 \times 10^0 = 20000 \times 1 = 20{,}000$

$\therefore$ Initial value $= \$20{,}000$

b. What is the value of the car after 5 years? (Give your answer to the nearest dollar)

When $t=5$, $V = 20000 \times 10^{-0.03 \times 5}$

$V = 20000 \times 10^{-0.15} \approx \$14{,}159$

c. When does the value of the car fall below $8,000? Give your answer in years correct to two decimal places.

$8000 = 20000 \times 10^{-0.03x}$

$\therefore x \approx 13.26$ years

Question 4

The population of a town, P, is modelled by the rule: $P = 15000 \times 2^{0.01t}$ where t is the number of years since 1996.

a. What was the population in 1996?

When $t=0$, $P = 15000 \times 2^0$

$= 15{,}000$

b. Find the population of the town in the year

i. 2000

In year 2000, $t = 14$ $\quad \therefore P = 15000 \times 2^{0.01 \times 14}$

$= 15000 \times 2^{0.14} = \cancel{165{,}286}$ $16{,}529$

ii. 2010

In year 2010, $t = 24$ $\quad P = 15000 \times 2^{0.24} = 17{,}715$

c. In what year will the population reach 20,000?

$$20000 = 15000 \times 2^{0.01t}$$

Solving: $t \simeq 41.5$

$\therefore$ During the year 2037.

d. According to this model, how long would it take for the population to double? Solve this WITHOUT using CAS.

$$30000 = 15000 \times 2^{0.01t}$$

$$\therefore \frac{30000}{15000} = 2^{0.01t}$$

$$2^1 = 2^{0.01t}$$

$$1 = 0.01t$$

$$t = \frac{1}{0.01}$$

$\therefore t = 100$ years

Question 5

The temperature T (°C) of a cooling cup of coffee in a room of temperature 20°C can be modelled by the rule: $T = 90 \times 3^{-kt}$, where t is the number of minutes after it is poured.

a. What is the temperature of the coffee when it is poured?

When $t = 0$, $T = 90 \times 3^0 = 90\,°C$

b. After 20 minutes, the temperature of the coffee is 30°. Use this information to find the value of k, without using CAS.

When $t = 20$, $T = 30$

$$T = 90 \times \cancel{30}\ 3^{-kt}$$

$$\therefore 30 = 90 \times 3^{-20k}$$

$$\therefore \frac{1}{3} = 3^{-20k}$$

$$3^{-1} = 3^{-20k}$$

$$k = \frac{1}{20}$$

c. Find the temperature, correct to two decimal places, of the coffee after:

i. 3 minutes $T = 90 \times 3^{-\frac{t}{20}}$

When $t = 3$, $T = 90 \times 3^{-3/20} \simeq 76.33\,°C$

ii. 6 minutes.

When $t = 6$, $T = 90 \times 3^{-6/20} \simeq 64.73\,°C$

d. How long does it take for the coffee to reach half its initial value? Give your answer correct to one decimal place.

$45 = 90 \times 3^{-\frac{t}{20}}$

Solving $t \simeq 12.6$ minutes

Question 6

Iodine is a radioactive isotope used in medicine. The concentration, C mg, remaining can be modelled by the equation $C = A_0 \times 2^{-kt}$ where A_0 represents the initial mass of the isotope (in mg), and t is the time in days. A patient is given 25 mg and 12.5 mg remains after 5 days.

(a) Determine the value of A_0 and k.

When $t = 0$, $C = 25$ $\therefore 25 = A_0 \times 2^0$

$\therefore A_0 = 25$

$C = 25 \times 2^{-kt}$

When $t = 5$, $C = 12.5$

$\therefore 12.5 = 25 \times 2^{-5k}$

$\frac{12.5}{25} = 2^{-5k}$

$\therefore \frac{1}{2} = 2^{-5k}$

$2^{-1} = 2^{-5k}$

$\therefore k = \frac{1}{5}$

(b) How many days will it take the concentration to reduce to 3.125 mg?

$C = 25 \times 2^{-\frac{t}{5}}$

When $C = 3.125$,

$3.125 = 25 \times 2^{-\frac{t}{5}}$

$\therefore \frac{3.125}{25} = 2^{-\frac{t}{5}}$

$\therefore \frac{1}{8} = 2^{-\frac{t}{5}}$

$2^{-3} = 2^{-\frac{t}{5}}$

$\therefore -3 = -\frac{t}{5}$

$\therefore t = 15.$

It will take 15 days