

SOLUTIONS

THE ADDITION LAW OF PROBABILITY WORKSHEET

Question 1

A jar contains 11 red, 13 blue 16 white and 10 yellow marbles. A marble is selected at random from the jar. What is the probability that it is either blue or white?

$$\begin{aligned} \Pr(B \cup W) &= \Pr(B) + \Pr(W) \\ &= \frac{13}{50} + \frac{16}{50} = \frac{29}{50} \end{aligned}$$

(B = selects blue
W = selects white
B & W are mutually exclusive).

Question 2

A set of cards is numbered {1, 2, 3 ... 16}.
A card is selected at random. Find:

(a) P(multiple of 5 or a multiple of 6)

$$\text{Multiples of 5} = \{5, 10, 15\}$$

$$\text{Multiples of 6} = \{6, 12, 18\}$$

$$\Pr(\text{Multiples of 6 or Multiples of 5}) = \frac{6}{16} = \frac{3}{8}$$

(b) P(a number less than 7 or greater than 8).

$$L = \{1, 2, 3, 4, 5, 6\}$$

$$G = \{9, 10, 11, 12, 13, 14, 15, 16\}$$

$$\Pr(L \cup G) = \frac{14}{16} = \frac{7}{8}$$

(c) Pr (a multiple of 3 or a multiple of 5)

$$T = \text{Multiples of 3} = \{3, 6, 9, 12, 15\}$$

$$F = \text{Multiples of 5} = \{5, 10, 15\}$$

$$T \cup F = \{3, 6, 9, 12, 15, 5, 10\}$$

$$\therefore \Pr(T \cup F) = \frac{7}{16}$$

Question 3

A card is selected at random from a pack of 52 playing cards. Find:

(a) P(a heart or a black card)

$$\begin{aligned} H &= \text{a heart} \\ B &= \text{a black card} \end{aligned}$$

$$\begin{aligned} \Pr(H \cup B) &= \Pr(H) + \Pr(B) - \Pr(H \cap B) \\ &= \frac{1}{4} + \frac{1}{2} - 0 \\ &= \frac{3}{4} \end{aligned}$$

(b) Pr (an Ace or a card from Clubs).

$$\begin{aligned} A &= \text{an Ace} \\ C &= \text{clubs} \end{aligned}$$

$$\begin{aligned} \Pr(A \cup C) &= \Pr(A) + \Pr(C) - \Pr(A \cap C) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} = \frac{4}{13} \end{aligned}$$

(c) Pr (Queen or a Red card)

$$\begin{aligned} Q &= \text{A queen} \\ R &= \text{A red card} \end{aligned}$$

$$\begin{aligned} \Pr(Q \cup R) &= \Pr(Q) + \Pr(R) - \Pr(Q \cap R) \\ &= \frac{4}{52} + \frac{1}{2} - \frac{2}{52} = \frac{28}{52} = \frac{14}{26} = \frac{7}{13} \end{aligned}$$

Question 4

In a chess tournament, the probability that Kevin Knight wins is $\frac{2}{11}$ and the probability that Bobby Bishop wins is $\frac{1}{44}$. What is the probability that either Kevin Knight or Bobby Bishop wins the tournament?

K = Kevin wins
 B = Bobby wins
 $K \cap B = \emptyset$

$$\begin{aligned} \Pr(K \cup B) &= \Pr(K) + \Pr(B) \\ &= \frac{2}{11} + \frac{1}{44} \\ &= \frac{8}{44} + \frac{1}{44} = \frac{9}{44} \end{aligned}$$

Question 5

In a harness race, the odds of two horses winning are, respectively, 5-1 and 2-1. What is the probability that either horse wins?

$$\begin{aligned} \Pr(\text{Horse 1 wins}) &= \frac{1}{6} & \Pr(\text{Horse 2 wins}) &= \frac{1}{3} \\ \Pr(\text{Horse 1 wins} \cup \text{Horse 2 wins}) &= \frac{1}{6} + \frac{1}{3} \\ &= \frac{1}{2} & \text{Required probability} &= \frac{1}{2} \end{aligned}$$

Question 6

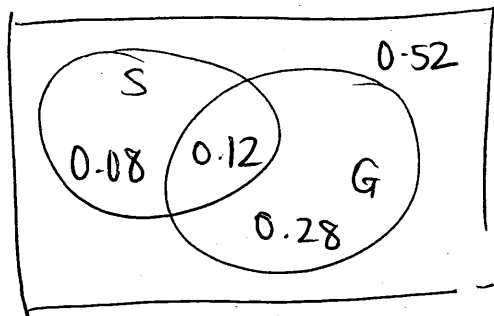
In a population of foxes, the probability that any randomly selected fox has a silver tip on its tail is 0.2. The probability that a randomly selected fox has grey eyes is 0.4. The probability that any randomly selected fox has grey eyes and a silver tip on its tail is 0.12.

- a. Find the probability that any fox selected randomly from this population has either a silver tip on its tail or grey eyes.

S = has silver tip
 G = has grey eyes

$$\begin{aligned} \Pr(S \cup G) &= \Pr(S) + \Pr(G) - \Pr(S \cap G) \\ \Pr(S \cup G) &= 0.2 + 0.4 - 0.12 \\ &= 0.48 \end{aligned}$$

- b. Illustrate this information on a Venn Diagram and calculate the probability that a Fox has grey eyes but no silver tip on its tail.



$$\Pr(G \cap S^c) = 0.28.$$

Question 7

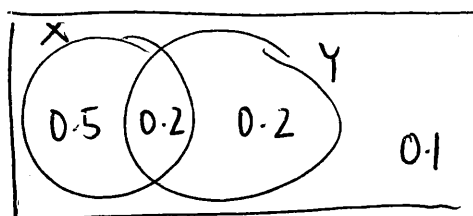
Given that: $\Pr(X) = 0.7$, $\Pr(Y) = 0.4$ and $\Pr(X \cup Y) = 0.9$:

- a. Calculate $\Pr(X \cap Y)$

$$\Pr(X \cup Y) = \Pr(X) + \Pr(Y) - \Pr(X \cap Y)$$

$$\therefore 0.9 = 0.7 + 0.4 - \Pr(X \cap Y) \quad \therefore \Pr(X \cap Y) = 0.2$$

- b. Illustrate this information on a Venn Diagram, and calculate $\Pr(X \cap Y')$



$$\Pr(X \cap Y') = 0.5$$

Question 8

At Nowhere University, the probability that a first year student studies Mathematics is 0.55. The probability that a first year student studies IT is 0.72. The probability that a first year student studies both Mathematics and IT is 0.4.

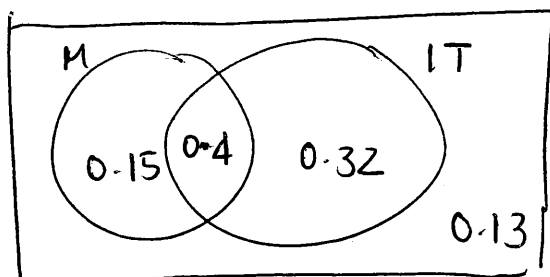
- a. Calculate the probability that a first year student from Nowhere University studies either Mathematics or IT.

$$\Pr(M \cup IT) = \Pr(M) + \Pr(IT) - \Pr(M \cap IT)$$

$$\Pr(M \cup IT) = 0.55 + 0.72 - 0.4 = 0.87 \quad \therefore \text{Required probability} = 0.87$$

- b. Illustrate on a Venn Diagram and calculate the probabilities that:

- A first year student studies Mathematics but not IT
- A first year student studies neither Mathematics nor IT.



$$(i) \Pr(M \cap IT') = 0.15$$

$$(ii) \Pr(M' \cap IT') = 0.13$$

Question 9

When a person contracts a type of influenza, the probability that he or she will either have a cough or a headache is 0.92. The probability that any person with this type of flu will have a headache is 0.8 and the probability that he or she will have a ~~headache~~ cough is 0.5. What is the probability that any person who gets this type of flu will have both a cough and a headache?

C = has a cough

H = has a headache

$$\Pr(H \cup C) = \Pr(H) + \Pr(C) - \Pr(H \cap C)$$

$$\therefore 0.92 = 0.8 + 0.5 - \Pr(H \cap C)$$

$$0.92 = 1.3 - \Pr(H \cap C) \quad \therefore \Pr(H \cap C) = 0.38$$

Required probability

$$= 0.38$$