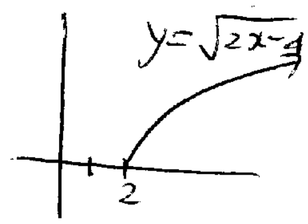
SOLUTIONS



Question 5

The inverse function of g: $[2, \infty) \to R$, $g(x) = \sqrt{2x-4}$ is

A. g^{-1}: $[2, \infty) \to R$, $g^{-1}(x) = \dfrac{x^2+4}{2}$

B. g^{-1}: $[0, \infty) \to R$, $g^{-1}(x) = (2x-4)^2$

C. g^{-1}: $[0, \infty) \to R$, $g^{-1}(x) = \sqrt{\dfrac{x}{2}+4}$

D. g^{-1}: $[0, \infty) \to R$, $g^{-1}(x) = \dfrac{x^2+4}{2}$

E. g^{-1}: $R \to R$, $g^{-1}(x) = \dfrac{x^2+4}{2}$

dom(g) $[2,\infty)$ ran(g) $[0,\infty)$

dom(g^{-1}) $[0,\infty)$ ran(g^{-1}) $[2,\infty)$

$y = \sqrt{2x-4}$

$x = \sqrt{2y-4}$

$\therefore x^2 = 2y-4$

$\therefore y = \dfrac{x^2+4}{2}, \ x \geq 0$

$\therefore g^{-1}(x) = \dfrac{x^2+4}{2}, \ x \in [0, \infty)$

Question 7

The inverse of the function f: $R^+ \to R$, $f(x) = \dfrac{1}{\sqrt{x}} - 3$ is

A. f^{-1}: $R^+ \to R$ $\qquad f^{-1}(x) = (x+3)^2$

B. f^{-1}: $R^+ \to R$ $\qquad f^{-1}(x) = \dfrac{1}{x^2} + 3$

C. f^{-1}: $(3, \infty) \to R$ $\qquad f^{-1}(x) = \dfrac{-1}{(x-3)^2}$

D. f^{-1}: $(-3, \infty) \to R$ $\qquad f^{-1}(x) = \dfrac{1}{(x+3)^2}$

E. f^{-1}: $(-3, \infty) \to R$ $\qquad f^{-1}(x) = -\dfrac{1}{x^2} - 3$

dom(g)	ran(g)
$(0,\infty)$	$(-3,\infty)$
dom(g^{-1})	ran(g^{-1})
$(-3,\infty)$	$(0,\infty)$

$y = \dfrac{1}{\sqrt{x}} - 3$

$\downarrow$

$x = \dfrac{1}{\sqrt{y}} - 3$

$\therefore x+3 = \dfrac{1}{\sqrt{y}} \ \therefore \ y = \dfrac{1}{(x+3)^2}, \ x > -3$

Question 9

The function f: $(-\infty, a] \to R$ with rule $f(x) = x^3 - 3x^2 + 3$ will have an inverse function provided

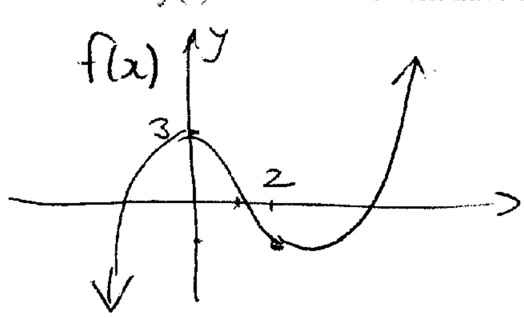
A. $a \leq 0$

B. $a \geq 2$

C. $a \geq 0$

D. $a \leq 2$

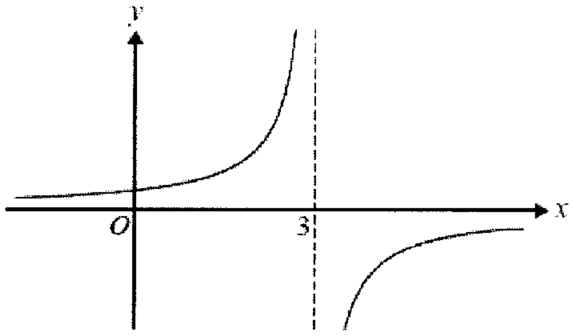
E. $a \leq 1$



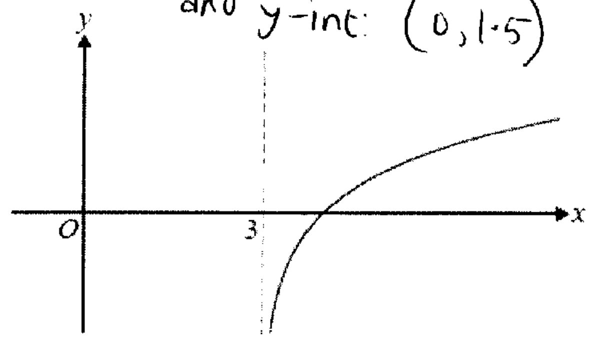
We need to restrict the domain of f so that it is one-to-one. From the graph, if $a = 0$ then f is one ~~the~~ ~~to~~ to one on domain $(-\infty, 0]$.

Only asymptote of inverse function is $x = 3$. Also has x-intercept at $(2, 0)$. and y-int: $(0, 1.5)$

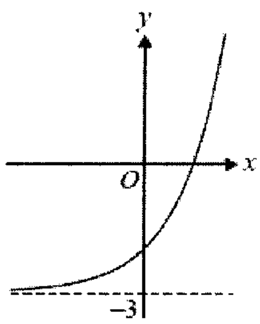
~~A.~~



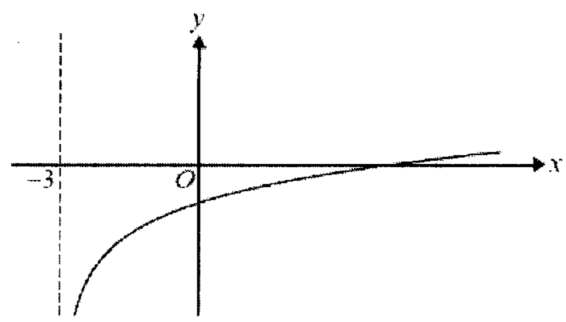
B.



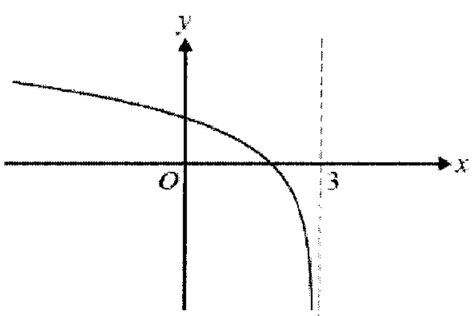
~~C.~~



~~D.~~



E.



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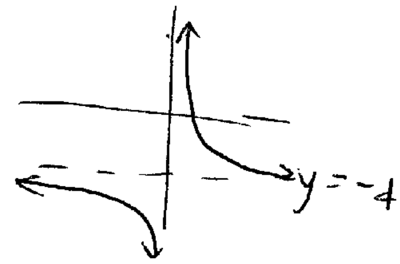


Question 3

Let $f: R\setminus\{0\} \to R$ where $f(x) = \frac{3}{x} - 4$. Find f^{-1}, the inverse function of f.

dom(f)	ran(f)
$\mathbb{R}\setminus\{0\}$	$\mathbb{R}\setminus\{-4\}$
dom(f^{-1})	ran(f^{-1})
$\mathbb{R}\setminus\{-4\}$	$\mathbb{R}\setminus\{0\}$

$x = \frac{3}{y} - 4$

$\therefore x + 4 = \frac{3}{y}$

$y = \frac{3}{x+4}$

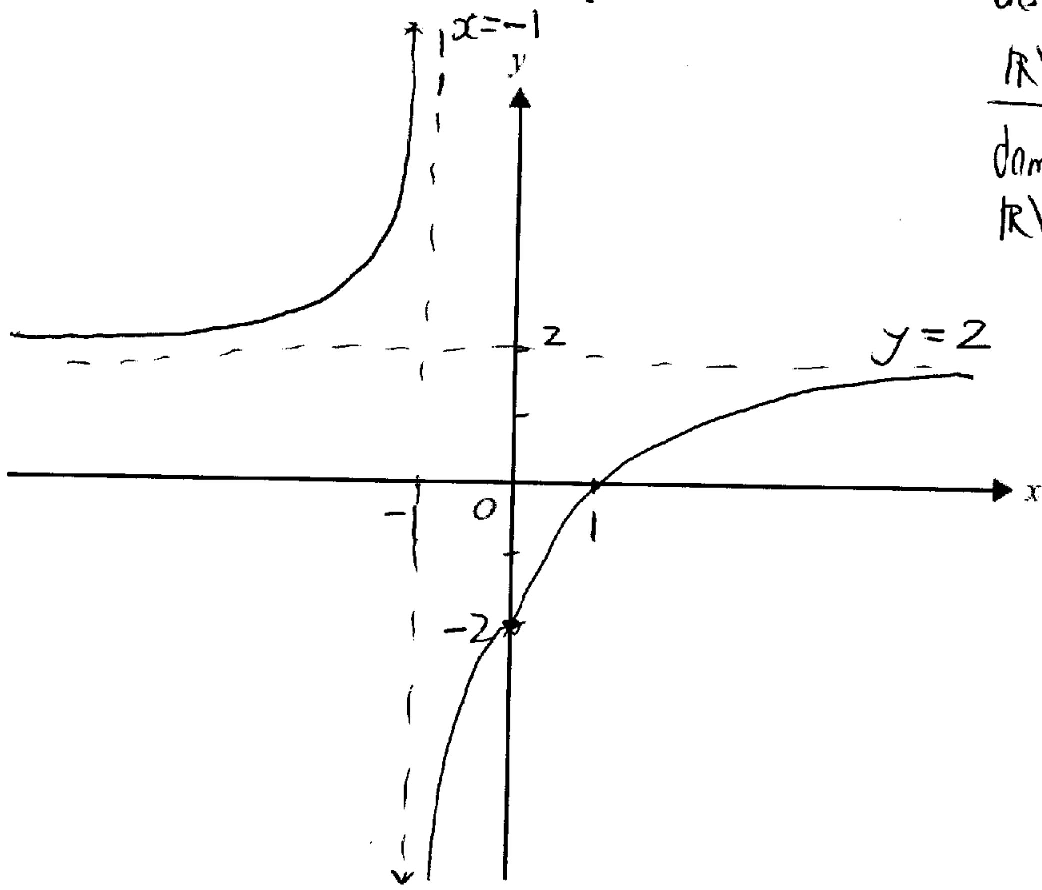
$\therefore f^{-1}(x) = \frac{3}{x+4}, \quad x \in \mathbb{R}\setminus\{-4\}.$

3 marks

Question 2

On the axes below, sketch the graph of $f: R\setminus\{-1\} \to R, \quad f(x) = 2 - \frac{4}{x+1}$.
Label all axis intercepts. Label each asymptote with its equation.

dom(f)	ran(f)
$\mathbb{R}\setminus\{-1\}$	$\mathbb{R}\setminus\{2\}$
dom(f^{-1})	ran(f^{-1})
$\mathbb{R}\setminus\{2\}$	$\mathbb{R}\setminus\{-1\}$



b. Find the rule of the inverse function f^{-1}, specifying its domain and range.

$x = 2 - \frac{4}{y+1}$

$\therefore x - 2 = \frac{-4}{y+1}$

$y + 1 = \frac{-4}{x-2}$

$\therefore f^{-1}(x) = \frac{-4}{x-2} - 1$

dom$(f^{-1}) = \mathbb{R}\setminus\{2\}$; ran$(f^{-1}) = \mathbb{R}\setminus\{-1\}$