 SOLUTIONS

Name: _____

MATHEMATICAL METHODS Unit 3
Test on Exponential and Logarithmic Functions **Time allowed: 40 minutes**

Section A Multiple Choice. *Circle the correct answer. (6 marks)*

1 Let the rule for a function g be $g(x) = \log_e\left((x-2)^2\right)$. For the function g, the

A maximal domain = R^+ and range = R
B maximal domain = $R \setminus \{2\}$ and range = R
C maximal domain = $R \setminus \{2\}$ and range = $(-2, \infty)$
D maximal domain = $[2, \infty)$ and range = $(0, \infty)$
E maximal domain = $[2, \infty)$ and range = $[0, \infty)$

$(x-2)^2 > 0$ if $x \in R \setminus \{2\}$

2 If $2\log_e x - \log_e 2x = q$ then x equals:

A 2
B e^{q^2}
C $2e^q$
D e^{2q}
E $\frac{1}{2}$

$\log_e x^2 - \log_e(2x) = q$
$\therefore \log_e\left(\frac{x^2}{2x}\right) = q$
$\log_e\left(\frac{x}{2}\right) = q \quad \therefore \frac{x}{2} = e^q$
$\therefore x = 2e^q$

3 The graph of $y = 6 - 5\log_e(x+2)$ has an asymptote at:

A $y = 6$
B $x = 2$
C $x = -2$
D $y = -5$
E $x = 6$

log functions only have vertical asymptotes!

4 The expression $\log_c(a) + \log_a(b) + \log_b(c)$ is equal to

A $\frac{1}{\log_c(a)} + \frac{1}{\log_a(b)} + \frac{1}{\log_b(c)}$
B $\frac{1}{\log_a(c)} + \frac{1}{\log_b(a)} + \frac{1}{\log_c(b)}$
C $-\frac{1}{\log_a(b)} - \frac{1}{\log_b(c)} - \frac{1}{\log_c(a)}$
D $\frac{1}{\log_a(a)} + \frac{1}{\log_b(b)} + \frac{1}{\log_c(c)}$
E $\frac{1}{\log_c(ab)} + \frac{1}{\log_b(ac)} + \frac{1}{\log_a(bc)}$

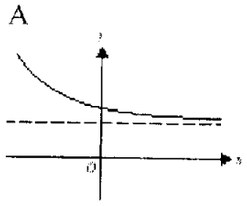
Change of base rule:
$\log_c a = \frac{\log_a a}{\log_a c} = \frac{1}{\log_a c}$

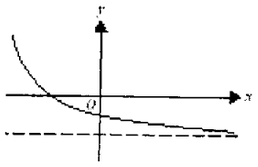
5 The function f has a rule $f(x) = 3\log_e(2x)$. If $f(5x) = \log_e(y)$ then y is equal to

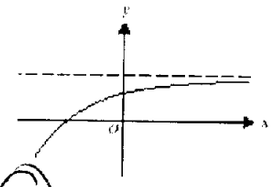
A $30x$
B $6x$
C $125x^3$
D $50x^3$
E $1000x^3$

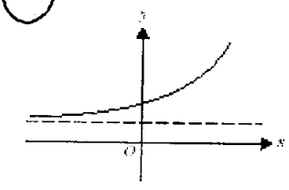
$f(5x) = 3\log_e(2x5x)$
$= 3\log_e(10x)$
$= \log_e\left((10x)^3\right)$
$\therefore y = 1000x^3$

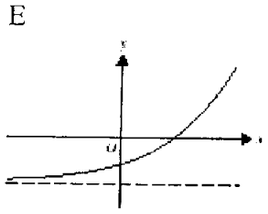
6 If k and P are positive real numbers, which one of the following graphs is most likely to be the graph of the function with equation $y = e^{kx} + P$:

A


B


C


D


E


Section B Short Answer *Show all working (16 marks)*

1 Simplify: $\dfrac{\log_a 16 - \log_a 2}{\log_a 2}$ 3 marks

$$\frac{\log_a 16 - \log_a 2}{\log_a 2}$$

$$= \frac{\log_a 2^4 - \log_a 2}{\log_a 2}$$

$$= \frac{4\log_a 2 - \log_a 2}{\log_a 2}$$

$$= \frac{3\log_a 2}{\log_a 2}$$

$$= 3$$

2 Solve for x in: $\log_3 2x + 4\log_3 6 - 2\log_3 12 = 4$ 4 marks

$$\log_3 2x + 4\log_3 6 - 2\log_3 12 = 4$$

$$\therefore \log_3\left(\frac{6^4 \times 2x}{12^2}\right) = 4$$

$$\log_3\left(\frac{6^2 \times 6^2 \times 2x}{6^2 \times 2^2}\right) = 4$$

$$\therefore \log_3(18x) = 4$$

$$18x = 3^4$$

$$x = \frac{81}{18}$$

$$\therefore x = \frac{9}{2}$$

3 Solve the equation $e^{3x} = x^2$ using your graphics calculator, giving your answer correct to 3 decimal places 1 mark

Solving on CAS: $x \approx -0.484$

4 Determine the equation of the inverse of $f(x) = 5\log_e 3x - 2$, stating its domain and range. 4 marks

$$y = 5\log_e 3x - 2$$

$$x = 5\log_e(3y) - 2$$

$$\frac{x+2}{5} = \log_e(3y)$$

$$e^{\frac{x+2}{5}} = 3y$$

$$y = \frac{1}{3}e^{\frac{x+2}{5}}$$

$$\therefore f^{-1}(x) = \frac{1}{3}e^{\frac{x+2}{5}}, \quad x \in \mathbb{R}$$

Range: $(0, \infty)$.

$\text{dom}(f)$	$\text{ran}(f)$
$(0, \infty)$	$\mathbb{R}$
$\text{dom}(f^{-1})$	$\text{ran}(f^{-1})$
$\mathbb{R}$	$(0, \infty)$

5 The graph of the function f with rule $f(x) = 2\log_e(x+3)+1$ intersects the axes at the points $(a, 0)$ and $(0, b)$ 4 marks

(a) Find the **exact** values of a and b

$0 = 2\log_e(a+3)+1$

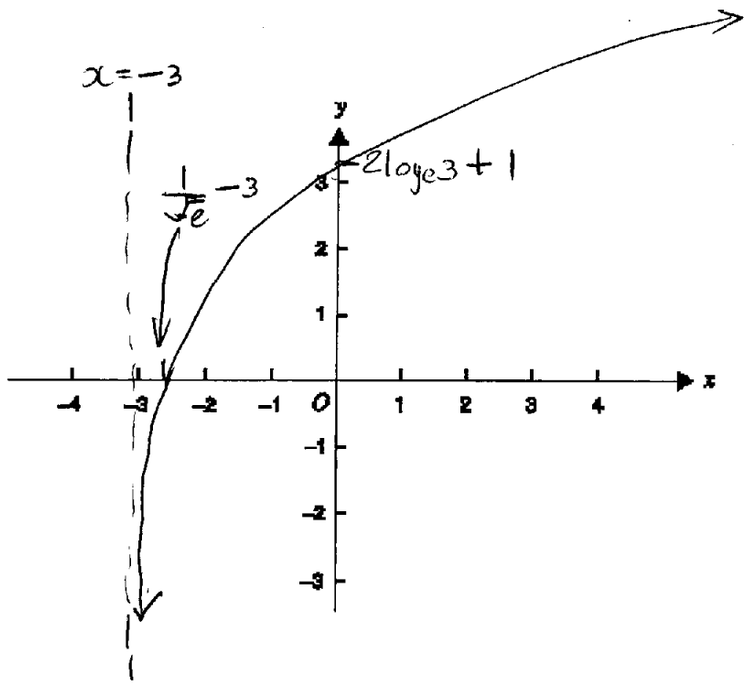
$-1 = 2\log_e(a+3)$

$e^{-\frac{1}{2}} = a+3$

$a = \frac{1}{\sqrt{e}} - 3$

$b = 2\log_e 3 + 1$

(b) Hence sketch the graph of the function with rule $f(x) = 2\log_e(x+3)+1$ on the axes below. Label any asymptote with its equation



Section C Extended Answer *Show all working (8 marks)*

1 Two colonies of starfish, P and Q, were studied and their populations found to be approximated by the models listed below:

Colony P: $P(t) = 60e^{0.1t}$

Colony Q: $Q(t) = 600 - 450e^{-0.1t}$

where t is the time in weeks after the start of the study.

(a) What was the population of each colony at the start of the study? 2 marks

$P(0) = 60e^0 = 60$

$Q(0) = 600 - 450e^0 = 150.$

(b) How long will it take for the population of colony P to double? Express your answer to the nearest week. 2 marks

$120 = 60e^{0.1t}$

$2 = e^{0.1t}$

$0.1t = \log_e 2$

$t = 10\log_e 2$

$t = 6.93$

$t \approx 7$ weeks

(c) By using the calculator, find to the nearest day, when the populations of the two starfish colonies are equal, and how many there are at this time. 2 marks

Define $p(t) = 60e^{0.1t}$ and $q(t) = 600 - 450e^{-0.1t}$

Solving $p(t) = q(t)$ gives $t = 22.1739$ (since $t > 0$)

$\therefore 22.1739 \times 7 \approx 155$ days

$p(22.1739) = q(22.1739) = 551$ Population $= 551$.

(d) Sketch the population of both colonies on the same axes, using an appropriate domain? 2 marks