SOLUTIONS.

Q1

$$2\cos(2x) = -\sqrt{3} \qquad 0 \leq x \leq \pi$$

$$\cos(2x) = -\frac{\sqrt{3}}{2} \qquad 0 \leq 2x \leq 2\pi$$

$$\therefore 2x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{5\pi}{12}, \frac{7\pi}{12}$$

Q2.

(a) $\frac{dy}{dx} = u'v + v'u$ $\qquad u = \log_e x \quad u' = \frac{1}{x}$

$\qquad\qquad\qquad\qquad\quad v = x^2 \quad v' = 2x$

$$\therefore \frac{dy}{dx} = \frac{1}{x} \times x^2 + 2x \log_e x$$

$$= x + 2x \log_e x$$

(b) $f(x) = e^{x^2}$

$$\therefore f'(x) = e^{x^2} \cdot 2x$$

$$= 2x e^{x^2}$$

$$f'(3) = 6e^9$$

Q3.

$g(x) = x^2 \sin(2x)$ $\quad u = x^2 \quad v = \sin(2x)$

$u' = 2x \quad v' = 2\cos(2x)$

$g'(x) = 2x^2\cos(2x) + 2x\sin(2x)$

$g'\left(\frac{\pi}{6}\right) = 2\times\left(\frac{\pi}{6}\right)^2\cos\left(\frac{\pi}{3}\right) + 2\times\frac{\pi}{6}\sin\left(\frac{\pi}{3}\right)$

$= \frac{\pi^2}{18}\cos\left(\frac{\pi}{3}\right) + \frac{\pi}{3}\times\frac{\sqrt{3}}{2}$

$= \frac{\pi^2}{36} + \frac{\sqrt{3}\pi}{6}$

Q4. $\sin\left(2x+\frac{\pi}{3}\right) = \frac{1}{2}, \quad 0 \le x \le \pi$

$0 \le 2x \le 2\pi$

$\frac{\pi}{3} \le 2x+\frac{\pi}{3} \le \frac{7\pi}{3}$

not in domain

$\therefore 2x+\frac{\pi}{3} = \cancel{\frac{\pi}{6}}, \frac{5\pi}{6}, \frac{13\pi}{6}$

$2x = \frac{5\pi}{6} - \frac{2\pi}{6}, \frac{13\pi}{6} - \frac{2\pi}{6}$

$2x = \frac{3\pi}{6}, \frac{11\pi}{6}$

$x = \frac{3\pi}{12}, \frac{11\pi}{12}$

$\therefore x = \frac{\pi}{4}, \frac{11\pi}{12}$

Q5(a) $f(x) = x^3 e^{2x}$

Let $u = x^3$, $u' = 3x^2$

$v = e^{2x}$, $v' = 2e^{2x}$

$f'(x) = x^3 . 2e^{2x} + e^{2x} . 3x^2$

$\therefore f'(x) = 2x^3 e^{2x} + 3x^2 e^{2x}$

(b) $f(x) = \log_e (x^2+1)$

$\therefore f'(x) = \frac{1}{x^2+1} \cdot 2x$

$= \frac{2x}{x^2+1}$

$\therefore f'(2) = \frac{2 \times 2}{2^2+1} = \frac{4}{5}$

Q6.

(a) $f(x) = e^{\cos 2x}$

$\therefore f'(x) = e^{\cos x} \cdot (-\sin x)$

$= -\sin x \, e^{\cos 2x}$

(b) $y = x \tan x$. Let $u = x$, $u' = 1$

$v = \tan x$, $v' = \sec^2 x$

$\frac{dy}{dx} = x \sec^2 x + \tan x$

$\frac{dy}{dx}\Big|_{x=\frac{\pi}{6}} = \frac{\pi}{6} \times \frac{1}{\cos^2(\frac{\pi}{6})} + \frac{1}{\sqrt{3}} = \frac{\pi}{6} \times \frac{4}{3} + \frac{1}{\sqrt{3}}$

$= \frac{2\pi}{9} + \frac{1}{\sqrt{3}}$