# SOLUTIONS (MULTIPLE CHOICE)

## Q1. C

$$2\cos\left(\frac{x}{2}\right)+\sqrt{3}=0$$

$$\cos\left(\frac{x}{2}\right)=-\frac{\sqrt{3}}{2}$$

$$\text{Period}=\frac{2\pi}{\frac{1}{2}}=4\pi$$

One solution: $\frac{x}{2}=\frac{5\pi}{6}$

$\therefore x=\frac{5\pi}{3}$

$\therefore$ General solution: $x=4n\pi\pm\frac{5\pi}{3}$, $n\in Z$

NB: If you use CAS, type in
solve $\left(\cos\left(\frac{x}{2}\right)=-\frac{\sqrt{3}}{2},x\right)$ without specifying a domain.
You get: $x=\frac{(12\cdot n1-5)\pi}{3}$ or $x=\frac{(12n1+5)\pi}{3}$

which is: $x=\frac{12n\pi}{3}-\frac{5\pi}{3}$ or $x=\frac{12n\pi}{3}+\frac{5\pi}{3}$

or $x=4n\pi-\frac{5\pi}{3}$ or $x=4n\pi+\frac{5\pi}{3}$

## Q2. D

$-b+4$

$-b$

$-b-4$

$$y=-4\sin\left(a(x+b)\right)-b$$

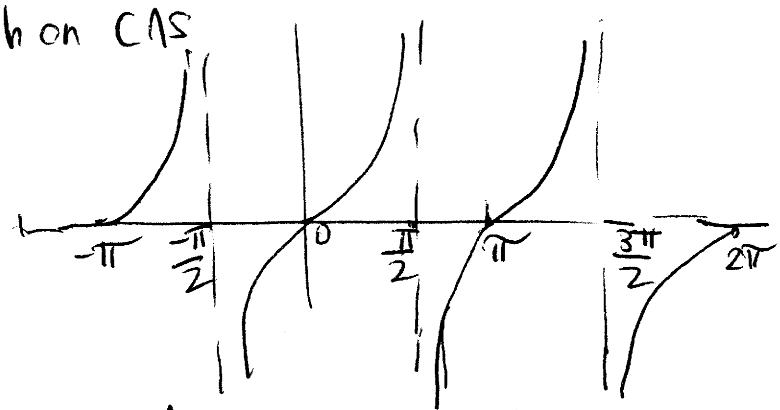
Amplitude $=4$

$\therefore$ Range: $[-b-4,\ -b+4]$

Q3. C

Asymptotes of $y = 2\pi\tan(x-\pi)+3$ are the same as for $y=\tan(x-\pi)$. Period $=\pi$

Graph on CAS




Asymptotes: $x=-\frac{\pi}{2},\ x=\frac{\pi}{2},\ x=\frac{3\pi}{2},$

Q4. D

$$\sin(x) \rightarrow \sin\left(x+\frac{\pi}{2}\right) \rightarrow \sin\left(x+\frac{\pi}{2}\right)+\frac{1}{2}$$

$$\rightarrow \sin\left(\frac{x}{2}+\frac{\pi}{2}\right)+\frac{1}{2}$$

$\therefore$ Transformed equation : $y=\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)+\frac{1}{2}$

$$\therefore y=\cos\left(\frac{x}{2}\right)+\frac{1}{2}$$

Q5 B

$$y = 2+\frac{4}{x-1}$$

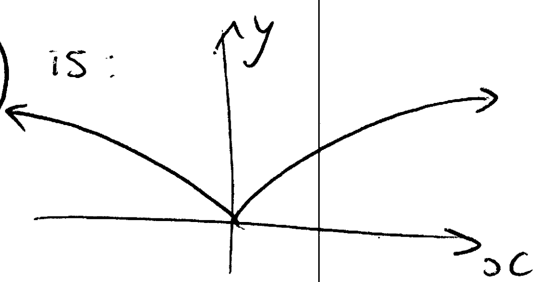
$$\begin{array}{r} 2 \\ x-1\overline{)2x+2} \\ \underline{2x-2} \\ 4 \end{array}$$

$\therefore$ domain: $\mathbb{R}\setminus\{1\}$

range: $\mathbb{R}\setminus\{2\}$

Q6. A

On CAS: $y = \log_e(2|x|+1)$ is:




NOTE: $2|x|+1 \geq 1$ for all $x$ so:

$\log_e(2|x|+1)$ is always defined.

$\therefore$ No asymptotes.

Q7. B

$(x-1)(\cancel{x}x^2+ax+b)=0$ has two unique solutions if: $x^2+ax+b=0$ has only one solution (which is not equal to 1).

$\therefore$ ~~x~~ $x^2+ax+b$ must be a perfect square and $\Delta = 0$

$\therefore$ ~~$b^2 \geq 4a \neq 0$ and~~

$\therefore$ $a^2 - 4b = 0$ and $a \neq -2$

$\therefore a^2 = 4b,\ a \neq -2$

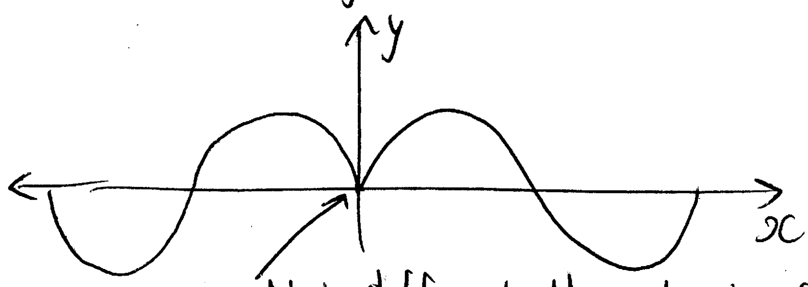
Q8. A

$g(f(x)) = \left(\sqrt{2+x}\right)^2 + 5$, $\operatorname{dom}(g(f(x))) = \operatorname{dom}(f(x)) = [-2, \infty)$

$\therefore g(f(x)) = x+7,\ x \in [-2, \infty)$

Q9. E

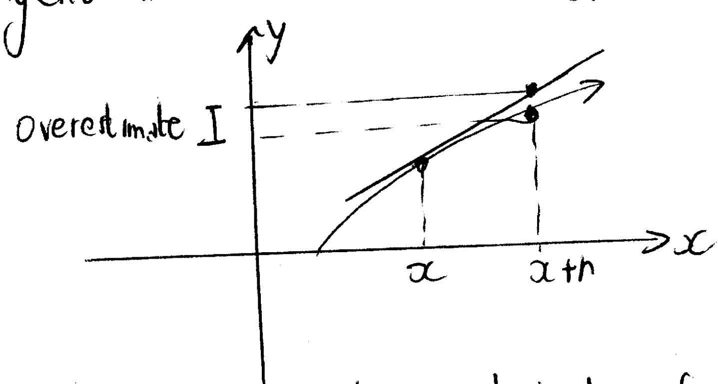
On CAS: $y = \sin(|x|)$



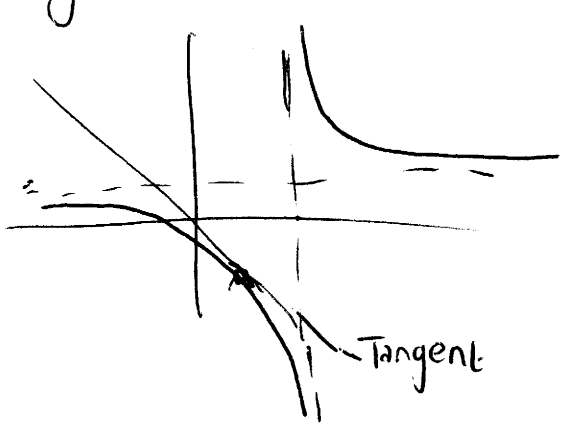

Not differentiable at $x = 0$ ("sharp point)

Q10. E

An overestimate for the approximation will occur when the tangent lies above the curve:




If you graph all the listed functions, the only one which has a section of the curve where the tangent lies above it would be E: $y = \frac{3}{x-2} + 1$




For all other functions, the tangents will always lie below the curve.

Q11. E

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left( \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \right)$$

$$= \lim_{h \to 0} \left( \frac{x+1-(x+h+1)}{h(x+h+1)(x+1)} \right)$$

$$= \lim_{h \to 0} \frac{-h}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+1)(x+h+1)}$$

$$= \lim_{h \to 0} \frac{-1}{x^2+hx+2x+h+1}$$

Q12. D

~~$y = x(x+1)^3$~~

Eliminate A and C, as these quartics could not have a stationary point of inflexion.

Eliminate B as: $f(1) = (1)^4 + 2(1)^3 - 1 + 2$

$= 1 + 2 - 1 + 2$

$= 4$

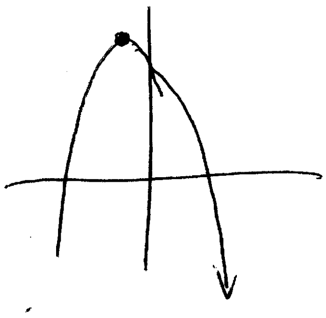
and according to the information, the graph must pass through $(1, 3)$.

In fact, it must be D because the cubed component is $(x+1)^3$, meaning a stationary point of inflexion when $x = -1$

Q13. C

$$g - f = -x^4 + x^2 + 7 - x^2 - 2x - 3$$
$$= -x^4 - 2x + 4$$

Graph $h(x) = -x^4 - 2x + 4$ on CAS



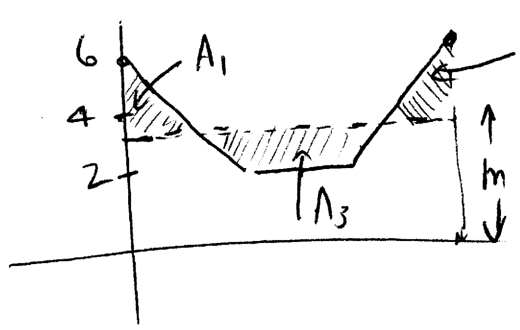
$$h'(x) = -4x^3 - 2$$
$$= 0 \text{ if}$$
$$4x^3 = -2$$
$$\therefore x = \sqrt[3]{-\frac{1}{2}}$$

Defining $h(x) = -x^4 - 2x + 4$,

$$h\left(\left(-\frac{1}{2}\right)^{1/3}\right) = \frac{3 \cdot 2^{\frac{2}{3}}}{4} + 4$$
$$= \frac{3 \cdot 2^{\frac{2}{3}}}{2^2} + 4$$
$$= \frac{3}{2^{4/3}} + 4$$

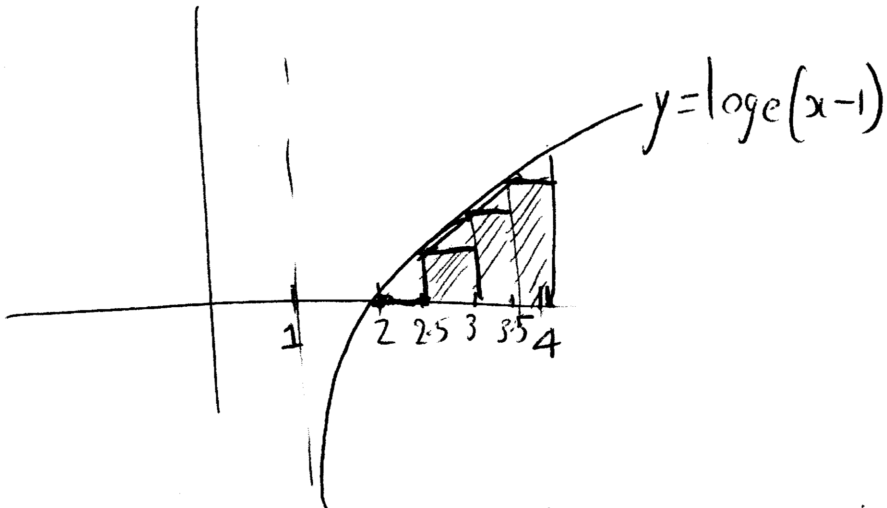
Q14. C

If $m$ = average value of function, then $m$ is the height of a rectangle whose area is the same as the area under the graph.




In other words, if $m$ is the average value of the function,
$A_1 + A_2 = A_3$
Clearly, the value of $m$ which makes sense here is $m = \frac{10}{3}$

Q15. A




$$A \approx 0.5 \times \log_e(2.5-1) + 0.5 \times \log_e(3-1) + 0.5 \times \log_e(3.5-1)$$

$$= \frac{1}{2}\log_e 1.5 + \frac{1}{2}\log_e 2 + \frac{1}{2}\log_e 2.5$$

$$= \frac{1}{2}\log_e\left(\frac{3}{2}\right) + \frac{1}{2}\log_e 2 + \frac{1}{2}\log_e\left(\frac{5}{2}\right)$$

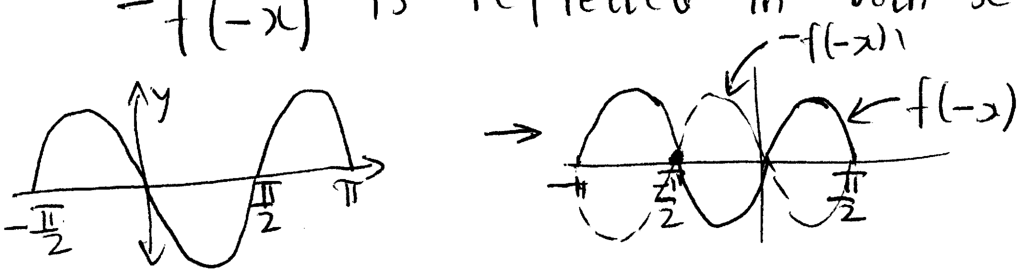
$$= \frac{1}{2}\log_e\left(\frac{3}{2} \times 2 \times \frac{5}{2}\right)$$

$$= \frac{1}{2}\log_e\left(\frac{15}{2}\right)$$

$$= \log_e\left(\sqrt{\frac{15}{2}}\right)$$

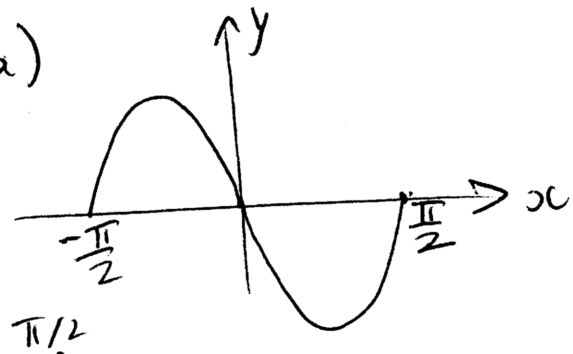
Q16.

$-f(-x)$ is reflected in both $x$- and $y$-axis:



Q16 (Cont) B

$-f(-x)$




$$\therefore \int_{-\pi/2}^{\pi/2} -f(-x)\,dx = A - A = 0$$

Q17. E

$$2p^2 + 0.1 + 0.02 + p + p^2 = 1$$

$$\therefore 3p^2 + p + \overset{0.12}{\cancel{1.02}} = 1$$

$$3p^2 + p - 0.88 = 0 \qquad \therefore p = 0.4$$

$$E(X^2) = 0^2 \times 2p^2 + 1^2 \times 0.1 + 2^2 \times 0.02 + 3^2 \times p + 4^2 \times p^2$$

$$= 0.1 + 0.08 + 9p + 16p^2$$

$$= 0.18 + 9p + 16p^2 \quad \therefore E(X^2) = 6.34$$

$$E(X) = 0 \times 2p^2 + 1 \times 0.1 + 2 \times 0.02 + 3p + 4p^2$$

$$= 0.14 + 3p + 4p^2$$

$$\therefore E(X) = 1.98$$

$$\therefore Var(X) = 6.34 - (1.98)^2 = 2.4196$$

$$\therefore \sigma = \sqrt{2.4196} = 1.55551$$

Q18. B

Let $X$ = no. of people selected

$X \overset{d}{=} Bi(n=50, p=0.05)$

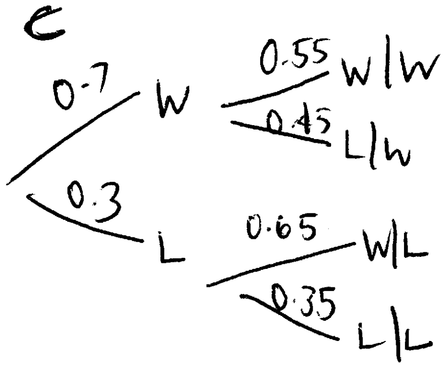
$X = 0, 1, 2, \ldots, 50$

$Pr(X<2) = Pr(X=0) + Pr(X=1)$

(use binomcdf with lower bound $=0$, and upper bound $=1$).

$= 0.2794$

Q19. C



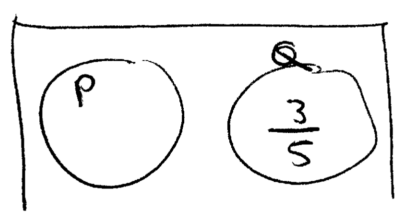

$\therefore T = \begin{bmatrix} 0.55 & 0.65 \\ 0.45 & 0.35 \end{bmatrix}$

$$\begin{bmatrix} 0.55 & 0.65 \\ 0.45 & 0.35 \end{bmatrix}^3 \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0.5908 \\ 0.4092 \end{bmatrix} \end{array} \begin{array}{l} W \\ L \end{array}$$

Q20. D

If P and Q are mutually exclusive, $Pr(P \cap Q) = 0$

$\therefore \frac{1}{2} Pr(P' \cap Q) = \frac{3}{10}$ $\therefore Pr(P' \cap Q) = \frac{3}{5}$



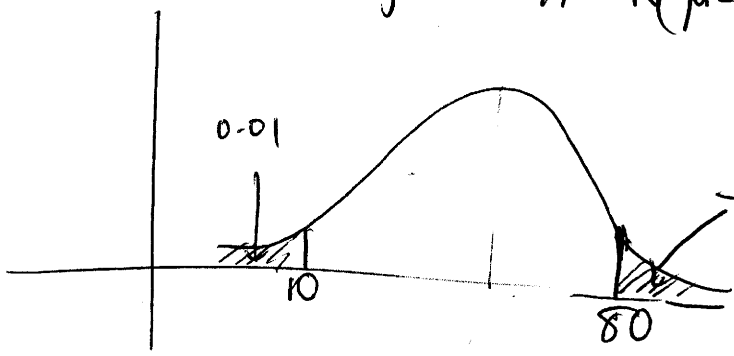

Note: if P, Q are mutually exclusive, $Pr(P' \cap Q) = Pr(Q)$

$\therefore Pr(P) + Pr(P' \cap Q') = \frac{2}{5}$

$\therefore Pr(P \cap Q') + Pr(P' \cap Q') = \frac{2}{5}$

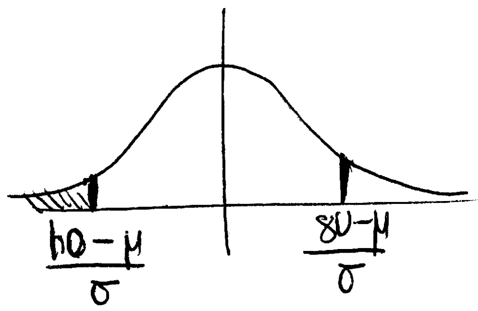
Q21. A

$H$ = height $\quad H \overset{d}{=} N(\mu = ?, \sigma = ?)$




$\Pr(H < 10) = 0.01$

$\Pr(H < 80) = \cancel{0.08}\ 0.915$




$\Pr\left(Z < \frac{10-\mu}{\sigma}\right) = 0.01$

$\Pr\left(Z < \frac{80-\mu}{\sigma}\right) = 0.915$

$\therefore \frac{10-\mu}{\sigma} = -2.32635 \quad$ (invNorm(0, 1, 0.01))

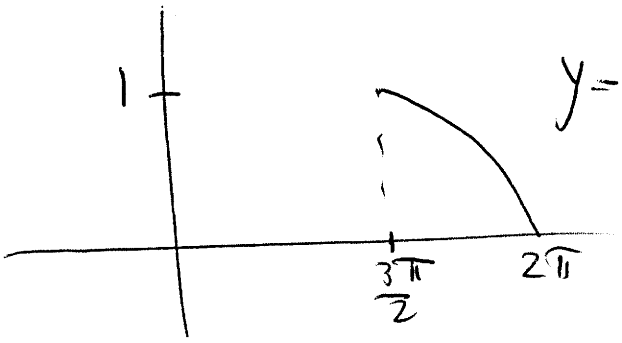
$\frac{80-\mu}{\sigma} = 1.3722 \quad$ (invNorm(0, 1, 0.915))

$\therefore \quad 10 - \mu = -2.32635\sigma$

$80 - \mu = 1.3722\sigma$

Solving: $\mu = 54.029, \ s = 18.93$

Q22. E




$$y = |\sin(x)|, \quad \frac{3\pi}{2} \le x \le 2\pi$$
$$= -\sin x, \quad \frac{3\pi}{2} \le x \le 2\pi$$

$$E(X^2) = \int_{3\pi/2}^{2\pi} x^2 |\sin x| \, dx$$
$$= \int_{3\pi/2}^{2\pi} -x^2 \sin x \, dx \approx 28.0536$$

$$E(X) = \int_{3/2}^{2\pi} -x \sin x \, dx \approx 5.28319$$

$$\therefore \operatorname{var}(X) = E(X^2) - [E(X)]^2$$
$$= 28.0536 - (5.28319)^2$$
$$= 0.1415$$