Question 5 (7 marks)

Consider the function $f:[-1,3]\to R,\ f(x)=3x^2-x^3$.

a. Find the coordinates of the stationary points of the function. 2 marks

$f(-1) = 3(-1)^2-(-1)^3 = 3+1=4 \quad \therefore (-1,4)$

$f(3) = 3\times(3)^2-3^3 = 0 \quad \therefore (3,0)$

$f'(x) = 6x-3x^2$

For stationary points, $f'(x)=0$

$\therefore 3x(2-x)=0$

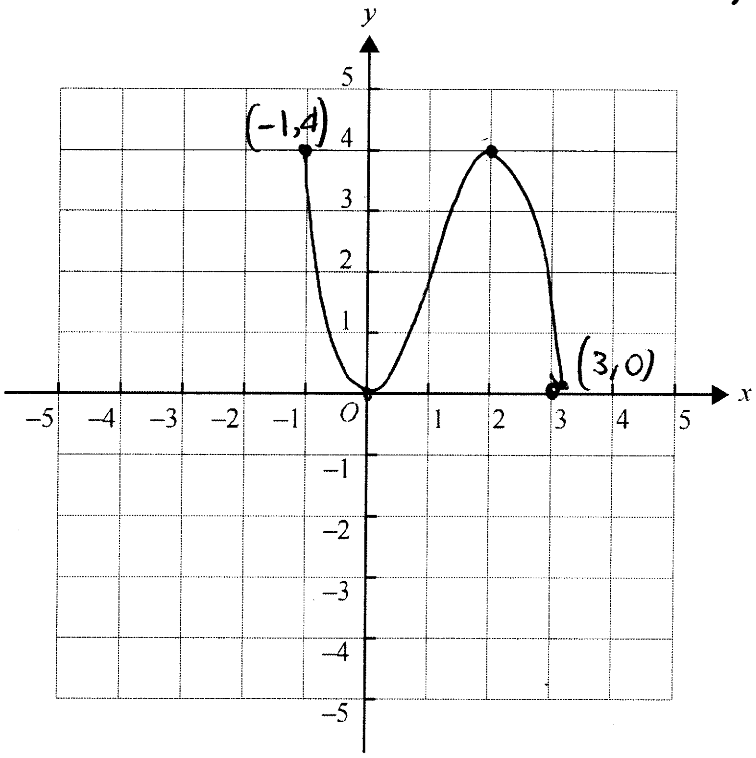
$x=0,2 \quad f(0)=0$

$f(2) = 3\times 2^2-2^3$

$= 12-8 = 4$

$\therefore (2,4)$

b. On the axes below, sketch the graph of f.
Label any end points with their coordinates. 2 marks



Question 2 (2 marks)

Solve $\log_e(x)-3=\log_e\left(\sqrt{x}\right)$ for x, where $x>0$.

$$\log_e x - \log_e \sqrt{x} = 3$$

$$\log_e x - \frac{1}{2}\log_e x = 3$$

$$\therefore \frac{1}{2}\log_e x = 3$$

$$\log_e x = 6$$

$$x = e^6$$

Question 3 (3 marks)

Let $f:(3,\infty)\to R$ where $f(x)=\log_e\left(\frac{x-3}{2}\right)+5$.

a. Find $f^{-1}(x)$.

dom (f)	ran (f)
$(3,\infty)$	$\mathbb{R}$
dom (f^{-1})	ran (f^{-1})
$\mathbb{R}$	$(3,\infty)$

$$x = \log_e\left(\frac{y-3}{2}\right)+5$$

$$x-5 = \log_e\left(\frac{y-3}{2}\right)$$

$$e^{x-5} = \frac{y-3}{2}$$

$$\therefore y = 2e^{x-5}+3$$

$$\therefore f^{-1}: \mathbb{R} \to \mathbb{R},\ f^{-1}(x) = 2e^{x-5}+3$$

b. Find $g(f(x))$, where $g(x)=f^{-1}(x)$. (2 marks)

$$f^{-1}(f(x)) = 3+2e^{\log_e\left(\frac{x-3}{2}\right)+5-5},\ \text{where } x>3$$

$$= 2e^{\log_e\left(\frac{x-3}{2}\right)} + 3$$

$$= 2\times\left(\frac{x-3}{2}\right)+3$$

$$= x-3+3 = x,\ x>3$$