Revision Sheet One

Question1

(8 marks)

Consider the functions $f(x) = 2\log_e(x)$ and $g(x) = x+1$

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a. State the maximal domain and range of $f(x) = 2\log_e(x)$ and $g(x) = x+1$

Domain of $f(x) = 2\log_e x$: $x \in (0, \infty)$

Domain of $g(x) = x+1$: $x \in \mathbb{R}$

Range of $f(x)$: $y \in \mathbb{R}$

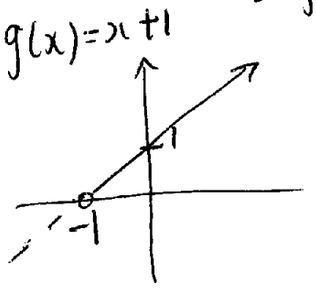
Range of $g(x)$: $y \in \mathbb{R}$

2 marks

b. if the domain of g is restricted to (a,∞), find the smallest value of a such that $f[g(x)]$exists. Find also, a rule for $f[g(x)]$.

$f[g(x)]$ exists if $\text{ran}(g) \subseteq \text{dom}(f)$

$\therefore \text{ran}(g) \subseteq (0, \infty)$.



We require: $g(x) > 0$

$\therefore x+1 > 0$

$\therefore x > -1$

$\therefore$ The domain of $g(x)$ must be restricted to $(-1, \infty)$

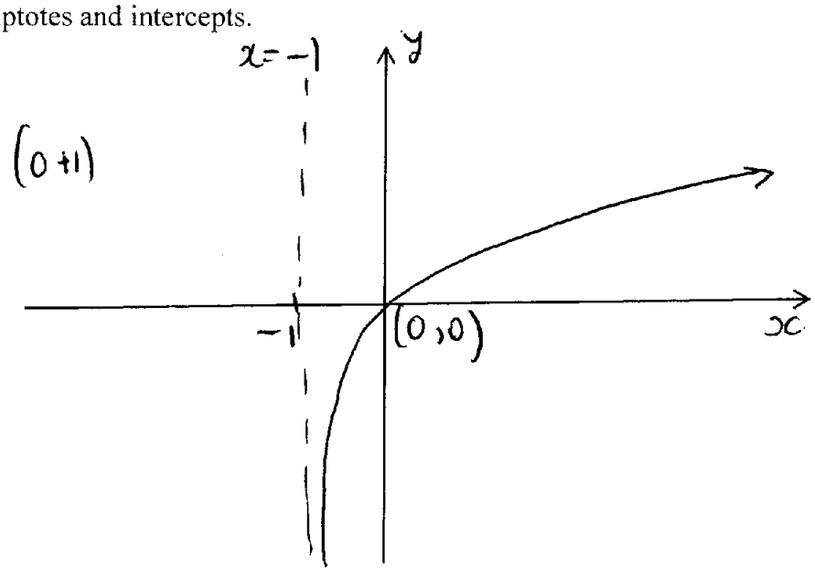
$\therefore a = -1$.

$f(g(x)) = 2\log_e(x+1),\ x \in (-1,\infty)$

2 marks

c. For this value of a, sketch the function $f[g(x)]$ on the axes below. Label any asymptotes and intercepts.

$f(0) = 2\log_e(0+1)$
$= 2\log_e 1$
$= 0.$



2 marks

d. Find the rule of $(f \circ g)^{-1}(x)$ and state its domain and range.

Let $(f \circ g) = y$

$y = 2\log_e(x+1)$

Interchange:

$x = 2\log_e(y+1)$

$\frac{x}{2} = \log_e(y+1)$

$\therefore y+1 = e^{\frac{x}{2}}$

$\therefore y = e^{\frac{x}{2}} - 1$

$\therefore (f \circ g)^{-1}(x) = e^{\frac{x}{2}} - 1$

Domain: $x \in \mathbb{R}$

Range: $(-1, \infty)$.

$\text{dom}(f \circ g)$	$\text{ran}(f \circ g)$
$(-1,\infty)$	$\mathbb{R}$
$\text{dom}((f \circ g)^{-1})$	$\text{ran}((f \circ g)^{-1})$
$\mathbb{R}$	$(-1,\infty)$

2 marks

Question 2 **(11 marks)**

Water is stored in a toilet cistern and when the flush button is pressed, water leaves the cistern and flushes the toilet bowl. Fresh water is then pumped back in to re-fill the cistern.
The height, h, in cm, of water in the cistern t seconds after the flush button is pressed is given by the function

$$h(t) = 12\cos\left(\frac{\pi t}{8}\right) + 12, \; t \in [0,16]$$

where $t = 0$ represents the time at which the flush button is pressed.

a. Write down

i. the amplitude of the function

Amplitude = 12

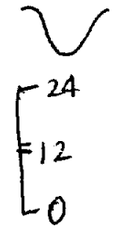
ii. the period of the function

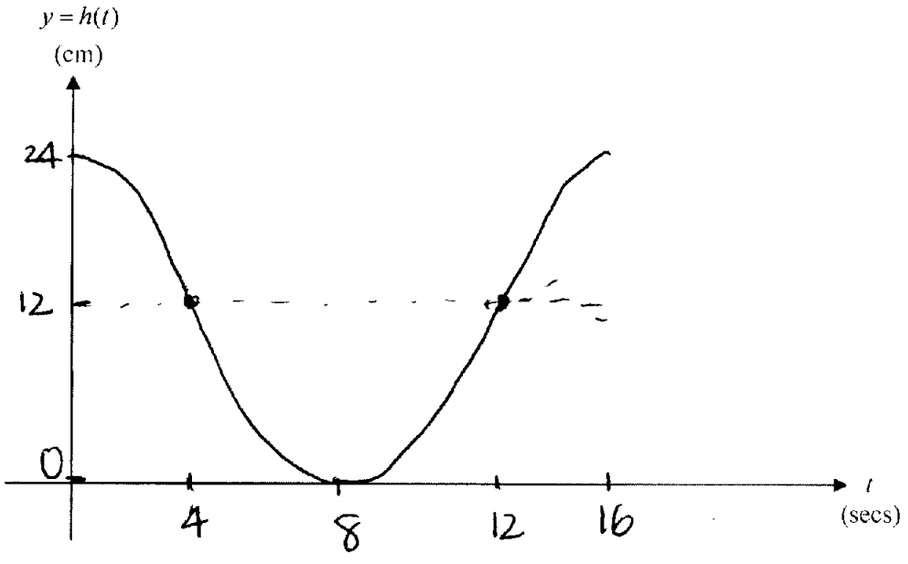
Period = $\frac{2\pi}{\frac{\pi}{8}} = 16$

1 + 1 = 2 marks

b. On the set of axes below, sketch the graph of the function $y = h(t)$. Show clearly on the graph any axes intercepts and the endpoints of the function.

Basic shape





2 marks

c. When is the cistern completely empty of water?

$t = 8$

1 mark

d. Between what times is fresh water being pumped back into the cistern?

Between $t = 8$ and $t = 16$.

1 mark

e. What is the maximum height that the water reaches in the cistern?

24 cm

1 mark

f. **i.** At what time did the height of the water first reach 18cm?

$18 = 12\cos\left(\frac{\pi t}{8}\right) + 12$

$\therefore \frac{1}{2} = \cos\left(\frac{\pi t}{8}\right)$ where $0 \leq t \leq 16$.

$\frac{\pi t}{8} = \frac{\pi}{3}, \frac{5\pi}{3}$ $\therefore t = \frac{8}{3}, \frac{40}{3}$ First time: $t = \frac{8}{3}$

ii. For what period of time during the period $t \in [0,16]$, was the height of the water in the cistern 18cm or less?

Required time interval $= \frac{40}{3} - \frac{8}{3}$

$= \frac{32}{3}$ seconds

2 + 2 = 4 marks

Total = 11 marks