MULTIPLE CHOICE REVISION

1

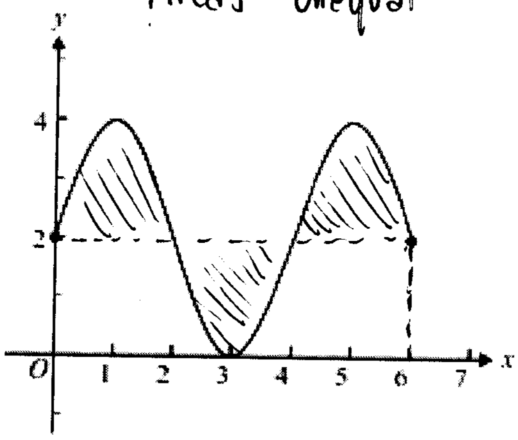
Question 15

Let h be a function with an average value of 2 over the interval [0, 6].

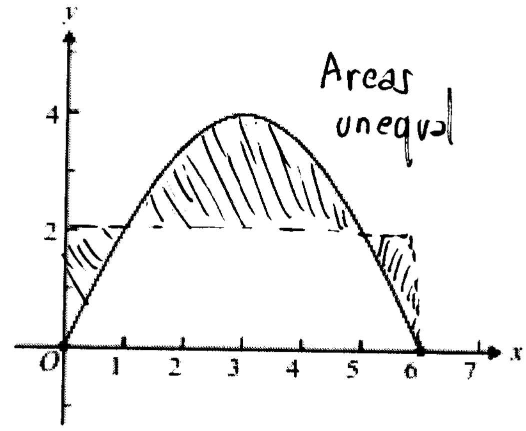
The graph of h over this interval could be

~~A.~~

Areas unequal

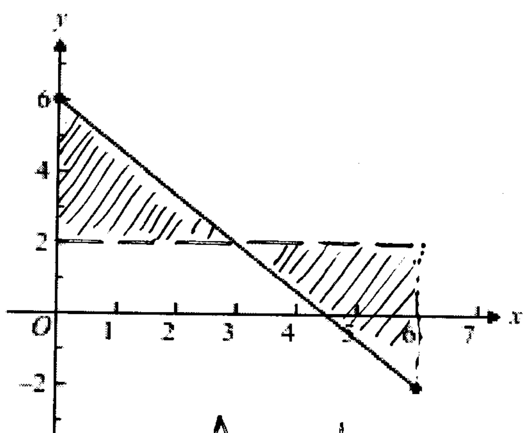


~~B.~~

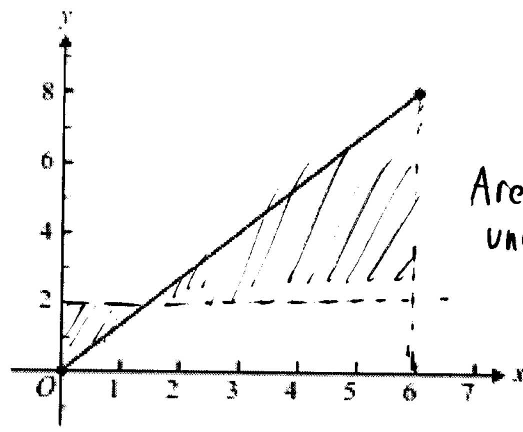


Areas unequal

C.



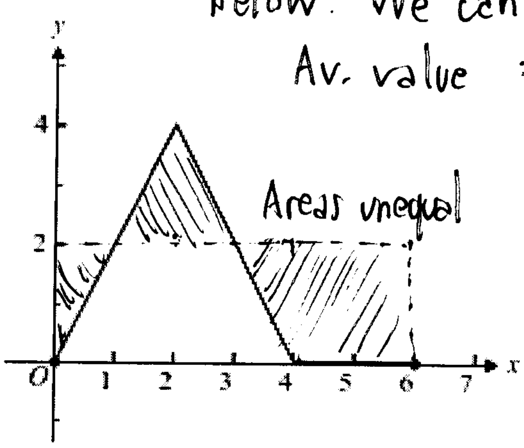
~~D.~~



Areas unequal

Area above $y=2$ is equal to area below. We can confirm this:

~~E.~~



Areas unequal

$$\text{Av. value} = \frac{1}{6}\int_0^6 f(x)\,dx \qquad m = \frac{-4}{3},\ c = 6$$

$$f(x) = \frac{-4x}{3} + 6$$

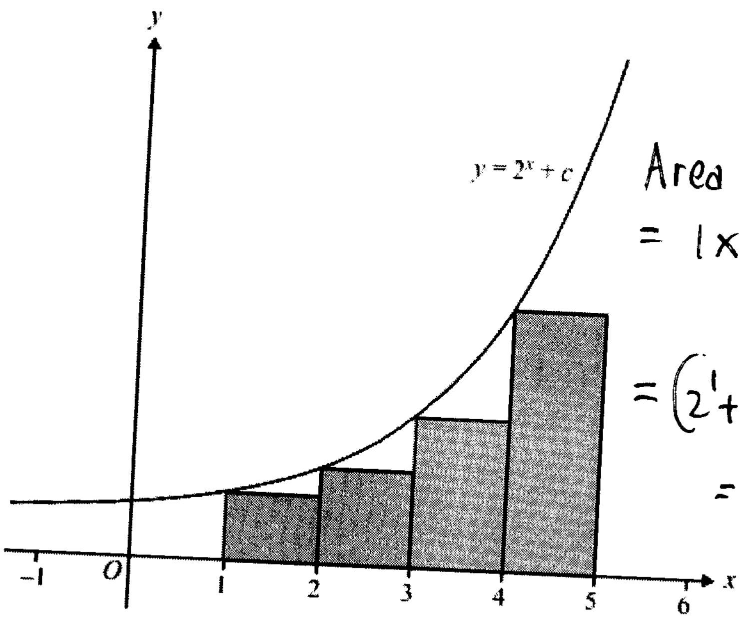
$$\frac{1}{6}\int_0^6 -\frac{4x}{3} + 6\,dx$$

$$= \frac{1}{6}\left[-\frac{2x^2}{3} + 6x\right]_0^6$$

$$= \frac{1}{6}(-24 + 36) = 2$$

Question 14

Consider the graph of $y = 2^x + c$, where c is a real number. The area of the shaded rectangles is used to find an approximation to the area of the region that is bounded by the graph, the x-axis and the lines $x = 1$ and $x = 5$.



Area of rectangles

$= 1\times f(1) + 1\times f(2) + 1\times f(3) + 1\times f(4)$

$= (2^1+c)+(2^2+c)+(2^3+c)+(2^4+c)$

$= 30 + 4c$

$\therefore 30 + 4c = 44$

$\therefore 4c = 14$

$c = \frac{7}{2}$

If the total area of the shaded rectangles is 44, then the value of c is

A. 14

B. –4

C. $\frac{14}{5}$

D. $\frac{7}{2}$

E. $-\frac{16}{5}$

Question 11

If the tangent to the graph of $y = e^{ax}$, $a \neq 0$, at $x = c$ passes through the origin, then c is equal to

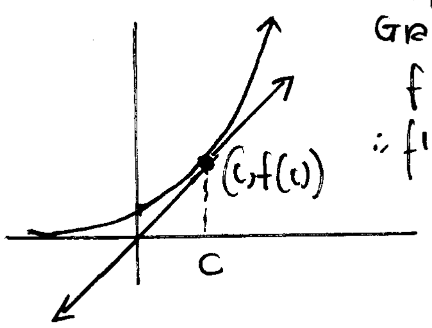
A. 0

B. $\frac{1}{a}$

C. 1

D. a

E. $-\frac{1}{a}$



Gradient of tangent $= f'(c)$

$f'(x) = ae^{ax}$

$\therefore f'(c) = ae^{ac}$

But gradient also equals

$\frac{f(c)-0}{c-0} = \frac{e^{ac}}{c}$

$\frac{e^{ac}}{c} = ae^{ac}$

$\therefore \frac{1}{c} = a$

$\therefore c = \frac{1}{a}$

Question 12

Let $y = 4\cos(x)$ and x be a function of t such that $\frac{dx}{dt} = 3e^{2t}$ and $x = \frac{3}{2}$ when $t = 0$.

The value of $\frac{dy}{dt}$ when $x = \frac{\pi}{2}$ is

A. 0

B. $3\pi\log_e\left(\frac{\pi}{2}\right)$

C. -4π

D. -2π

E. $-12e$

$\frac{dx}{dt} = 3e^{2t}$

$\therefore x = \int 3e^{2t}\,dt$

$= \frac{3}{2}e^{2t} + c$

When $t=0$, $x = \frac{3}{2}$

$\therefore \frac{3}{2} = \frac{3}{2}e^{0} + c$

$\therefore c = 0$

$\therefore x = \frac{3}{2}e^{2t}$

$\therefore y = 4\cos\left(\frac{3}{2}e^{2t}\right)$

$\frac{dy}{dt} = -4\sin\left(\frac{3}{2}e^{2t}\right) \times 3e^{2t}$

If $x = \frac{\pi}{2}$, then $\frac{3}{2}e^{2t} = \frac{\pi}{2}$

$\therefore 3e^{2t} = \pi$

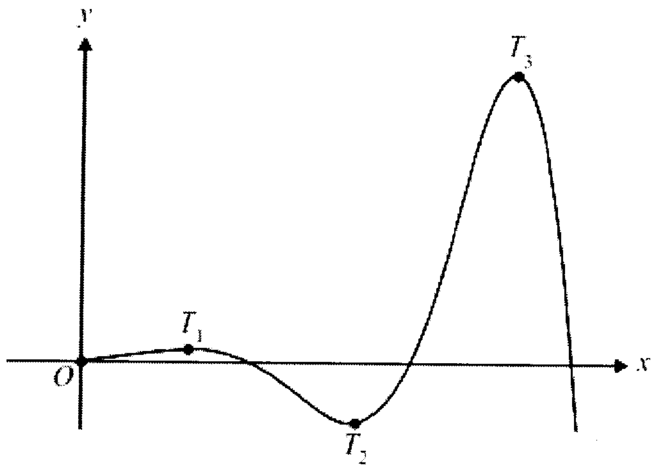
When $x = \frac{\pi}{2}$, $\frac{dy}{dt} = -4\sin\left(\frac{\pi}{2}\right) \times \pi$

$= -4\pi$

Question 19

Part of the graph of a function $f: [0, \infty) \to R, f(x) = e^{x\sqrt{3}} \sin(x)$ is shown below.
The first three turning points are labelled T_1, T_2 and T_3.

3



The x-coordinate of T_3 is

A. $\frac{8\pi}{3}$

B. $\frac{16\pi}{3}$

C. $\frac{13\pi}{6}$

D. $\frac{17\pi}{6}$

E. $\frac{29\pi}{6}$

$f(x) = e^{x\sqrt{3}} \sin x$

$\therefore f'(x) = e^{x\sqrt{3}} \cos x + \sqrt{3} e^{x\sqrt{3}} \sin x$

$= e^{x\sqrt{3}}(\cos x + \sqrt{3} \sin x)$

$f'(x) = 0$ if $\cos x + \sqrt{3} \sin x = 0$

$\therefore \sqrt{3} \sin x = -\cos x$

$\tan x = -\frac{1}{\sqrt{3}}$ $\therefore x = \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$

where $x > 0$

Question 20

Let f be a differentiable function defined for all real x, where $f(x) \geq 0$ for all $x \in [0, a]$.

If $\int_0^a f(x)dx = a$, then $2\int_0^{5a}(f(\frac{x}{5}) + 3)dx$ is equal to:

A. $2a + 6$

B. $10a+6$

C. $20a$

D. $40a$

E. $50a$

$2\int_0^{5a}(f(\frac{x}{5}) + 3)dx$

$= 2\int_0^{5a} f(\frac{x}{5})dx + \int_0^{5a} 6\,dx$

$= 2 \times 5a + [6x]_0^{5a} = 10a + 30a = 40a$

$f(\frac{x}{5})$ is a dilation of factor 5 away from x-axis

$\therefore \int_0^{5a} f(\frac{x}{5})dx = 5 \times \int_0^a f(x)dx$

$= 5a$

Question 21

The cubic function $f: R \to R, f(x) = ax^3 - bx^2 + cx$, where a, b and c are positive constants, has no stationary points when

A. $c > \frac{b^2}{4a}$

B. $c < \frac{b^2}{4a}$

C. $c < 4b^2a$

D. $c > \frac{b^2}{3a}$

E. $c < \frac{b^2}{3a}$

$f'(x) = 3ax^2 - 2bx + c$

For stationary points, $f'(x) = 0$

$3ax^2 - 2bx + c = 0$

This quadratic has no solution if $\Delta < 0$

$\therefore (-2b)^2 - 4(3a)(c) < 0$

$4b^2 - 12ac < 0$ $\therefore 4b^2 < 12ac$

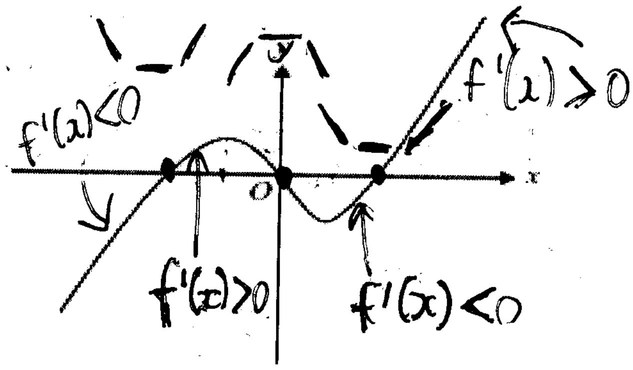
$4b^2 - 12ac < 0$

$\therefore 12ac > 4b^2$

$c > \frac{b^2}{3a}$

Question 19

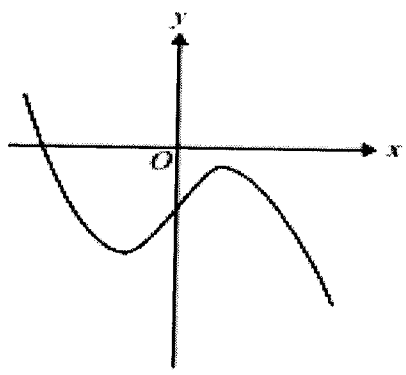
The graph of the gradient function $y = f'(x)$ is shown below.



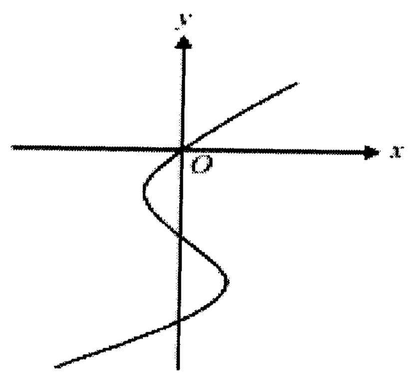
f(x) has three turning points, which are: min, max, min in that order.

Which of the following could represent the graph of the function $f(x)$?

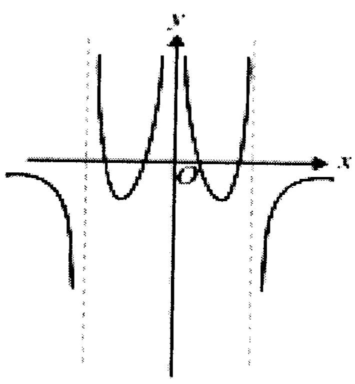
A.



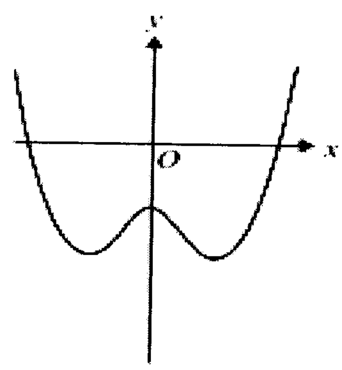
B.



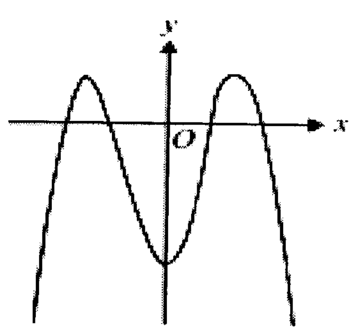
C.



D.



E.



Question 22

Let f be a differentiable function defined for $x > 2$ such that

$\int_3^{ab+2} f(x)dx = \int_3^{a+2} f(x)dx + \int_3^{b+2} f(x)dx$ where $a > 1$ and $b > 1$

A log function has the property $f(mn) = f(m) + f(n)$ So I guessed that we needed $f(x)$ to be the derivative of a log function.

The rule for $f(x)$ is:

A. $\sqrt{x-2}$

B. $\log_e(x-2)$

C. $\sqrt{2x-4}$

D. $\log_e|2x-4|$

E. $\frac{1}{x-2}$

$\int_3^{ab+2} \frac{1}{x-2}\, dx = \left[\log_e(x-2)\right]_3^{ab+2}$

$= \log_e(ab+2-2) - \log_e 1$

$= \log_e(ab)$

$\int_3^{a+2} f(x)dx + \int_3^{b+2} f(x)dx = \left[\log_e(x-2)\right]_3^{a+2} + \left[\log_e(x-2)\right]_3^{b+2}$

$= \log_e(a) - \log_e 1 + \log_e(b) - \log_e 1$

$= \log_e(a) + \log_e(b)$

$= \log_e(ab)$

Question 17

The function f is differentiable for all $x \in R$ and satisfies the following conditions.

- $f'(x) < 0$ where $x < 2$
- $f'(x) = 0$ where $x = 2$
- $f'(x) = 0$ where $x = 4$
- $f'(x) > 0$ where $2 < x < 4$
- $f'(x) > 0$ where $x > 4$

$-$ $+$ 2 $+$ 4 $+$

Which one of the following is true?

A. The graph of f has a local maximum point where $x = 4$.

B. The graph of f has a stationary point of inflection where $x = 4$.

C. The graph of f has a local maximum point where $x = 2$.

D. The graph of f has a local minimum point where $x = 4$.

E. The graph of f has a stationary point of inflection where $x = 2$.

Question 6

A function g with domain R has the following properties.

- $g'(x) = x^2 - 2x$
- the graph of $g(x)$ passes through the point $(1, 0)$

$g(x)$ is equal to

A. $2x - 2$

B. $\frac{x^3}{3} - x^2$

C. $\frac{x^3}{3} - x^2 + \frac{2}{3}$

D. $x^2 - 2x + 2$

E. $3x^3 - x^2 - 1$

$g(x) = \int x^2 - 2x\, dx$

$= \frac{x^3}{3} - x^2 + c$

$g(1) = 0$

$\therefore 0 = \frac{1}{3} - 1 + c$

$\therefore c = \frac{2}{3}$

$g(x) = \frac{x^3}{3} - x^2 + \frac{2}{3}$

Question 10

The average value of the function $f(x) = e^{2x}\cos(3x)$ for $0 \le x \le \pi$ is closest to

A. -82.5

B. 26.3

C. -26.3

D. -274.7

E. π

$$\frac{1}{\pi}\int_0^{\pi} e^{2x}\cos(3x)\,dx$$

Question 16

The gradient of the function $f: R \to R, f(x) = \dfrac{5x}{x^2+3}$ is negative for

A. $-\sqrt{3} < x < \sqrt{3}$

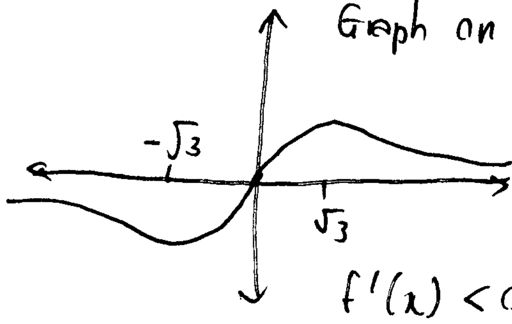
B. $x > 3$

C. $x \in R$

D. $x < -\sqrt{3}$ and $x > \sqrt{3}$

E. $x < 0$

Graph on CAS



$f'(x) < 0$ for

$x < -\sqrt{3} \cup x > \sqrt{3}$

Question 17

The normal to the curve with equation $y = x^{\frac{3}{2}} + x$ at the point (4,12) is parallel to the straight line with equation:

A. $4x = y$

B. $4y + x = 7$

C. $y = \dfrac{x}{4} + 1$

D. $x - 4y = -5$

E. $4y + 4x = 20$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 1$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{3}{2} \times \sqrt{4} + 1$$
$$= 4$$

$$\therefore m_{NORMAL} = -\frac{1}{4}$$

Option B is Correct since:

$$4y + x = 7$$
$$4y = -x + 7$$
$$y = -\frac{x}{4} + \frac{7}{4}$$
$$m = -\frac{1}{4}$$

Question 7

By considering the point (8, 2) on the graph of $f(x) = \sqrt[3]{x}$, and using the linear approximation method, $f(8+h) \approx f(8) + hf'(8)$, an estimate for $\sqrt[3]{8.5}$ is

A. 2.5000

B. 2.08333

C. 2.04167

D. 2.04083

E. 2.00347

$$f(x+h) \approx f(x) + h\,f'(x)$$

$$\therefore f(8+0.5) \approx f(8) + 0.5 \times f'(8)$$

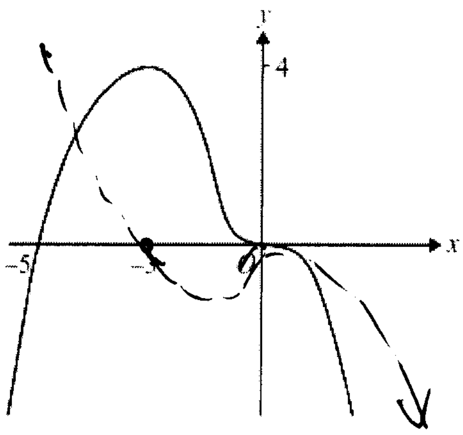
$$f'(x) = \frac{1}{3}x^{-2/3} \qquad \therefore f'(8) \approx \frac{1}{3} \times 8^{-2/3}$$
$$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$\therefore \sqrt[3]{8.05} \approx \sqrt[3]{8} + \frac{1}{2} \times \frac{1}{12} = 2 + \frac{1}{24}$$
$$= 2.04167$$

7

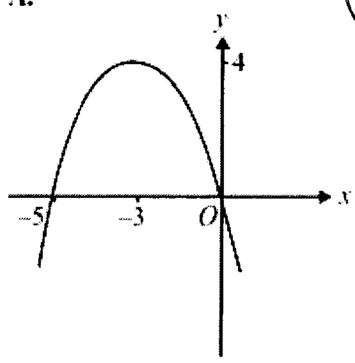
Question 9

The graph of the function $y = f(x)$ is shown below.

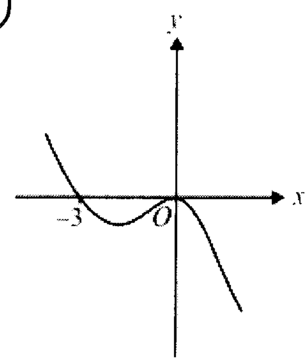


Which of the following could be the graph of the derivative function $y = f'(x)$?

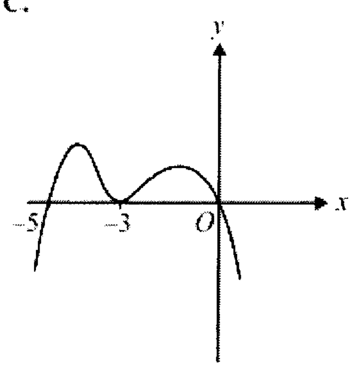
A.



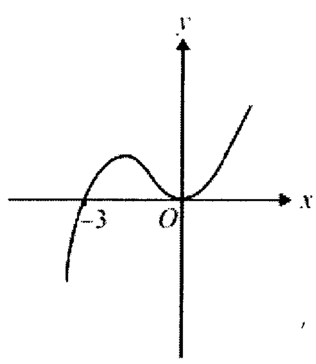
B.



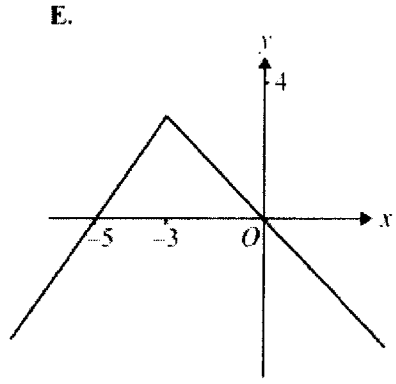
C.



D.

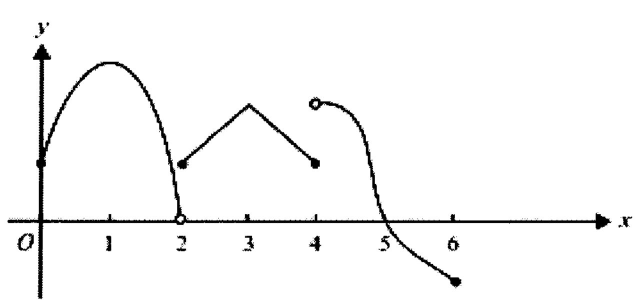


E.



Question 22

The graph of the function f with domain $[0, 6]$ is shown below.



Which one of the following is **not** true?

A. The function is not continuous at $x = 2$ and $x = 4$. True

$f(2) \neq 0$

B. The function exists for all values of x between 0 and 6. True

C. $f(x) = 0$ for $x = 2$ and $x = 5$.

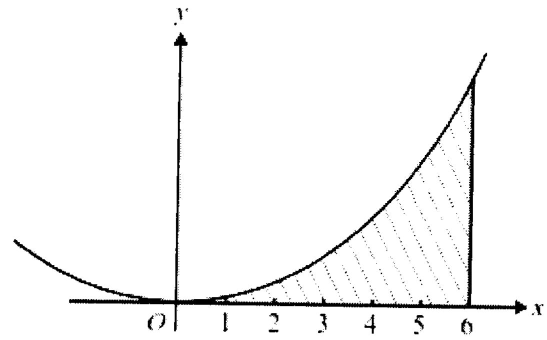
D. The function is positive for $x \in [0, 5)$. True

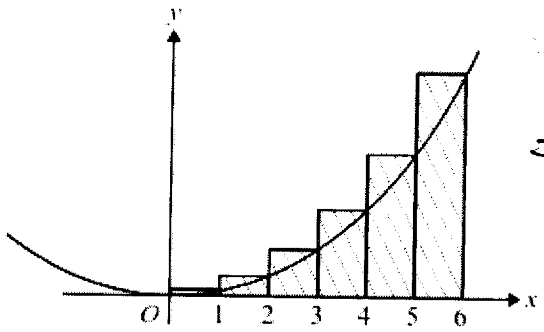
E. The gradient of the function is not defined at $x = 4$. True

(8)

Question 19

A part of the graph of $f : R \to R, f(x) = x^2$ is shown below. Zoe finds the approximate area of the shaded region by drawing rectangles as shown in the second diagram.





$$\text{Exact area} = \int_0^6 x^2\,dx$$
$$= \left[\frac{x^3}{3}\right]_0^6$$
$$= \frac{216}{3} = 72$$

Area of rectangles
$$= f(1)+f(2)+f(3)+f(4)+f(5)+f(6)$$
$$= 1^2+2^2+3^2+4^2+5^2+6^2$$
$$= 1+4+9+16+25+36$$
$$= 91$$

Over estimate $= 91 - 72 = 19$

$$\text{Percentage} = \frac{19}{72} \times \frac{100}{1}$$
$$\approx 26\%$$

Zoe's approximation is p% more than the exact value of the area.
The value of p is closest to

A. 10
B. 15
C. 20
D. 25
E. 30

Question 4

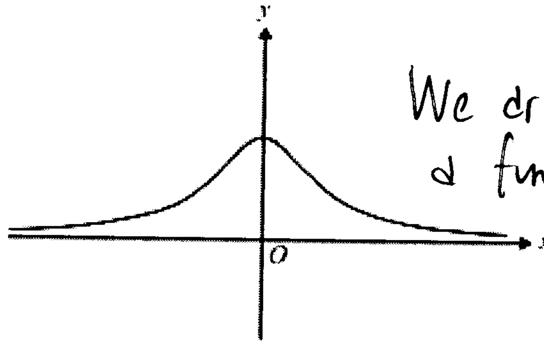
If $\int_1^3 f(x)dx = 5$, then $\int_1^3 (2f(x) - 3)dx$ is equal to

A. 4
B. 5
C. 7
D. 10
E. 16

$$\int_1^3 (2f(x) - 3)\,dx$$
$$= \int_1^3 2f(x)\,dx - \int_1^3 3\,dx$$
$$= 2\int_1^3 f(x)\,dx - [3x]_1^3$$
$$= 2 \times 5 - [(9) - (3)]$$
$$= 4$$

Question 19

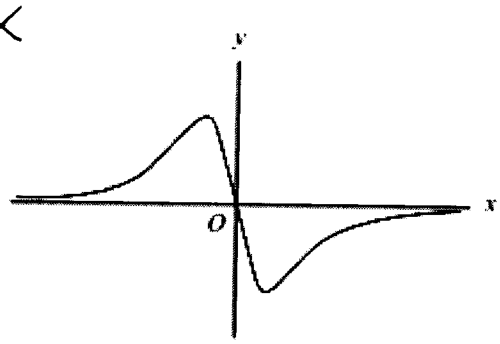
The graph of a function f is shown below.



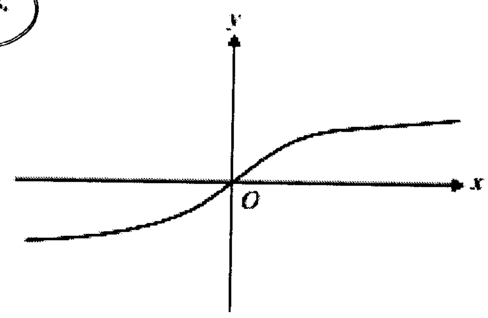
We are looking for a functions whose gradient is positive for all values of x, and is steepest at $x=0$

The graph of an **antiderivative** of f could be

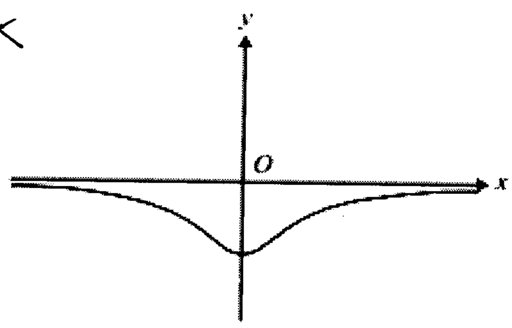
~~A.~~



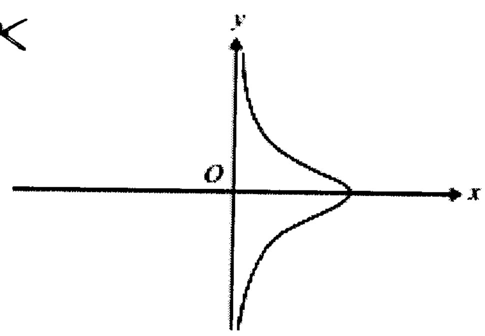
B.



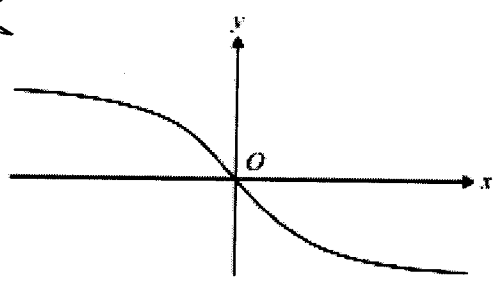
~~C.~~



~~D.~~



~~E.~~



Question 21

The volume, V cm^3, of water in a container is given by $V = \frac{1}{3}\pi h^3$ where h cm is the depth of water in the container at time t minutes. Water is draining from the container at a constant rate of 300 cm^3/min. The rate of decrease of h, in cm/min, when $h = 5$ is

A. $\frac{12}{\pi}$

B. $\frac{4}{\pi}$

C. 25π

D. $\frac{60}{\pi}$

E. 30π

$$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$$

$$\frac{dV}{dt} = -300$$

$$V = \frac{1}{3}\pi h^3$$

$$\frac{dV}{dh} = \pi h^2 \quad \therefore \frac{dh}{dV} = \frac{1}{\pi h^2}$$

$$\therefore \frac{dh}{dt} = -300 \times \frac{1}{\pi h^2}$$

When $h = 5$,

$$\frac{dh}{dt} = \frac{-300}{\pi \times 25} = \frac{-12}{\pi}$$