SOLVING SIMULTANEOUS EQUATIONS USING MATRICES

We have seen that if we have the simultaneous equations set up in matrix form:

$$A.X = K$$

(where *A* is the **coefficient matrix**)

then matrix *X* can be found using: $\boldsymbol{X = A^{-1}.K}$

provided that det(*A*) $\neq$ 0

Example 6 Solving a set of three simultaneous equations using the inverse matrix

Solve using matrix methods:

$$\begin{aligned} 3x + 4y - 2z &= -5 \\ 2x + 3y &= -1 \\ x + 2y + 3z &= 3 \end{aligned}$$

EXAMPLE 2:

Example 7 A practical application

A manufacturer makes two sorts of orange-flavoured chocolates: House Brand and Orange Delights. The number of kilograms of House Brand (x) and the number of kilograms of Orange Delights (y) that can be made from 80 kg of chocolate and 120 kg of orange filling can be found by solving the following pair of equations:

$$\begin{aligned} 0.3x + 0.5y &= 80 \\ 0.7x + 0.5y &= 120 \end{aligned}$$

Solve for x and y using matrix methods.

WHEN det(A) = 0

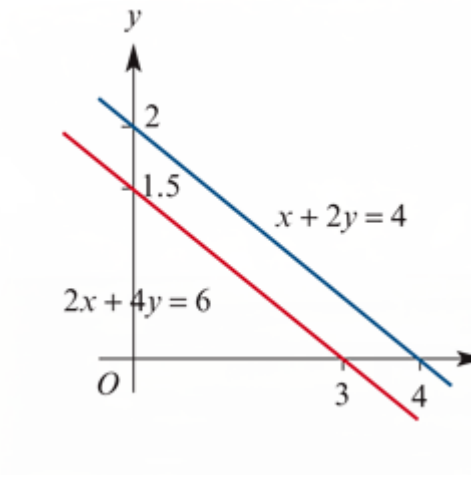
When the coefficient matrix A is singular, it means that the system of simultaneous equations has NO UNIQUE SOLUTION.

There are two possible cases:

<table>
<tr><th colspan="2">det(A) = 0</th></tr>
<tr><td>Inconsistent equations
NO SOLUTION
(parallel lines)</td><td>Dependent equations
INFINITELY MANY SOLUTIONS
(same line)</td></tr>
<tr><td>The pair of simultaneous equations:

$$x + 2y = 4$$
$$2x + 4y = 6$$

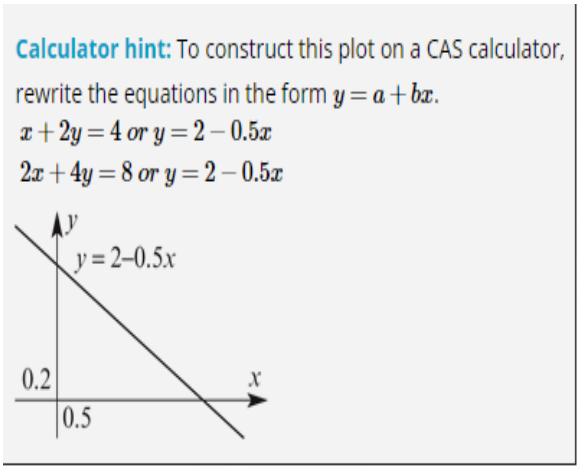
has NO solution because when graphed, they represent parallel lines

There is NO intersection point. Therefore, no solution.</td><td>The pair of simultaneous equations:

$$x + 2y = 4$$
$$2x + 4y = 8$$

has INFINITELY MANY solutions because in reality both equations represent the same line.

There are infinitely many intersection points, the entire line.</td></tr>
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Either way, if det(A) = 0, then the simultaneous equations do NOT have a unique (single) solution.