Make sure you keep referring to this as we progress throughout the year.

Mathematical Methods Units 3 and 4

Mathematical Methods Units 3 and 4 are completely prescribed and extend the introductory study of simple elementary functions of a single real variable, to include combinations of these functions, algebra, calculus, probability and statistics, and their applications in a variety of practical and theoretical contexts. Units 3 and 4 consist of the areas of study 'Functions and graphs', 'Calculus', 'Algebra' and 'Probability and statistics', which must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. Assumed knowledge and skills for Mathematical Methods Units 3 and 4 are contained in Mathematical Methods Units 1 and 2, and will be drawn on, as applicable, in the development of related content from the areas of study, and key knowledge and skills for the outcomes of Mathematical Methods Units 3 and 4.

For Unit 3 a selection of content would typically include the areas of study 'Functions and graphs' and 'Algebra', and applications of derivatives and differentiation, and identifying and analysing key features of the functions and their graphs from the 'Calculus' area of study. For Unit 4, this selection would typically consist of remaining content from the areas of study: 'Functions and graphs', 'Calculus' and 'Algebra', and the study of random variables and discrete and continuous probability distributions and the distribution of sample proportions. For Unit 4, the content from the 'Calculus' area of study would be likely to include the treatment of anti-differentiation, integration, the relation between integration and the area of regions specified by lines or curves described by the rules of functions, and simple applications of this content.

The selection of content from the areas of study should be constructed so that there is a development in the complexity and sophistication of problem types and mathematical processes used (modelling, transformations, graph sketching and equation solving) in application to contexts related to these areas of study. There should be a clear progression of skills and knowledge from Unit 3 to Unit 4 in each area of study.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations, graphs, differentiation, anti-differentiation, integration and inference with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Tick off each of these dotpoints as we cover them.

Functions and graphs

In this area of study students cover transformations of the plane and the behaviour of some elementary functions of a single real variable, including key features of their graphs such as axis intercepts, stationary points, points of inflection, domain (including maximal, implied or natural domain), co-domain and range, asymptotic behaviour and symmetry. The behaviour of these functions and their graphs is to be linked to applications in practical situations.

This area of study includes:

- graphs and identification of key features of graphs of the following functions:
  - power functions, $y = x^n$, $n \in Q$
  - exponential functions, $y = a^x$, $a \in R^+$, in particular $y = e^x$, and logarithmic functions, $y = \log_e(x)$ and $y = \log_{10}(x)$
  - circular functions, $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$
- graphs of polynomial functions
- transformation from $y = f(x)$ to $y = Af(n(x + b)) + c$, where A, n, b and $c \in R$, $A, n \neq 0$, and f is one of the functions specified above, and the inverse transformation

- the relation between the graph of an original function and the graph of a corresponding transformed function (including families of transformed functions for a single transformation parameter)
- graphs of sum, difference, product and composite functions where f and g are functions of the types specified above (not including composite functions that result in reciprocal or quotient functions), use of polynomial, power, circular, exponential and logarithmic functions, simple transformation and combinations of these functions, including simple piecewise (hybrid) functions, to model practical situations.

Area of Study 2

Algebra

In this area of study students cover the algebra of functions, including composition of functions, simple functional relations, inverse functions and the solution of equations. They also study the identification of appropriate solution processes for solving equations, and systems of simultaneous equations, presented in various forms. Students also cover recognition of equations and systems of equations that are solvable using inverse operations or factorisation, and the use of graphical and numerical approaches for problems involving equations where exact value solutions are not required or which are not solvable by other methods. This content is to be incorporated as applicable to the other areas of study.

This area of study includes:

- review of algebra of polynomials, equating coefficients and solution of polynomial equations with real coefficients of degree n having up to n real solutions
- use of simple functional relations such as $f(x + k) = f(x), f(x^n) = nf(x), f(x) + f(-x) = 0, f(xy) = f(x)f(y)$, to characterise properties of functions including periodicity and symmetry, and to specify algebraic equivalence, including the exponent and logarithm laws
- functions and their inverses, including conditions for the existence of an inverse function, and use of inverse functions to solve equations involving exponential, logarithmic, circular and power functions
- composition of functions, where f composition g is defined by $f(g(x))$, given $r_g \subseteq d_f$ (the notation $f \circ g$ may be used, but is not required)
- solution of equations of the form $f(x) = g(x)$ over a specified interval, where f and g are functions of the type specified in the 'Functions and graphs' area of study, by graphical, numerical and algebraic methods, as applicable
- solution of literal equations and general solution of equations involving a single parameter
- solution of simple systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist (geometric interpretation only required for two equations in two variables).

Area of Study 3

Calculus

In this area of study students cover graphical treatment of limits, continuity and differentiability of functions of a single real variable, and differentiation, anti-differentiation and integration of these functions. This material is to be linked to applications in practical situations.

This area of study includes:

- review of average and instantaneous rates of change, tangents to the graph of a given function and the derivative function
- deducing the graph of the derivative function from the graph of a given function and deducing the graph of an anti-derivative function from the graph of a given function

- derivatives of x^n, for $n \in Q$, e^x, $\log_e(x)$, $\sin(x)$, $\cos(x)$ and $\tan(x)$
- derivatives of $f(x) \pm g(x)$, $f(x) \times g(x)$, $\dfrac{f(x)}{g(x)}$ and $f(g(x))$ where f and g are polynomial functions, exponential, circular, logarithmic or power functions and transformations or simple combinations of these functions
- application of differentiation to graph sketching and identification of key features of graphs, identification of intervals over which a function is constant, stationary, strictly increasing or strictly decreasing, identification of the maximum rate of increase or decrease in a given application context (consideration of the second derivative is not required), identification of local maximum/minimum values over an interval and application to solving problems, and identification of interval endpoint maximum and minimum values
- anti-derivatives of polynomial functions and functions of the form $f(ax + b)$ where f is x^n, for $n \in Q$, e^x, $\sin(x)$, $\cos(x)$ and linear combinations of these
- informal consideration of the definite integral as a limiting value of a sum involving quantities such as area under a curve, including examples such as distance travelled in a straight line and cumulative effects of growth such as inflation
- anti-differentiation by recognition that $F'(x) = f(x)$ implies $\int f(x)dx = F(x) + c$
- informal treatment of the fundamental theorem of calculus, $\int_a^b f(x)dx = F(b) - F(a)$
- properties of anti-derivatives and definite integrals
- application of integration to problems involving finding a function from a known rate of change given a boundary condition, calculation of the area of a region under a curve and simple cases of areas between curves, distance travelled in a straight line, average value of a function and other situations.

Area of Study 4

Probability and statistics

In this area of study students cover discrete and continuous random variables, their representation using tables, probability functions (specified by rule and defining parameters as appropriate); the calculation and interpretation of central measures and measures of spread; and statistical inference for sample proportions. The focus is on understanding the notion of a random variable, related parameters, properties and application and interpretation in context for a given probability distribution.

This area of study includes:

- random variables, including the concept of a random variable as a real function defined on a sample space and examples of discrete and continuous random variables
- discrete random variables:
  - specification of probability distributions for discrete random variables using graphs, tables and probability mass functions
  - calculation and interpretation and use of mean (μ), variance (σ^2) and standard deviation of a discrete random variable and their use
  - bernoulli trials and the binomial distribution, $\text{Bi}(n, p)$, as an example of a probability distribution for a discrete random variable
  - effect of variation in the value/s of defining parameters on the graph of a given probability mass function for a discrete random variable
  - calculation of probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability

- continuous random variables:
  - construction of probability density functions from non-negative functions of a real variable
  - specification of probability distributions for continuous random variables using probability density functions
  - calculation and interpretation of mean (μ), median, variance (σ^2) and standard deviation of a continuous random variable and their use
  - standard normal distribution, $\mathrm{N}(0, 1)$, and transformed normal distributions, $\mathrm{N}(\mu, \sigma^2)$, as examples of a probability distribution for a continuous random variable
  - effect of variation in the value/s of defining parameters on the graph of a given probability density function for a continuous random variable
  - calculation of probabilities for intervals defined in terms of a random variable, including conditional probability (the cumulative distribution function may be used but is not required)
- Statistical inference, including definition and distribution of sample proportions, simulations and confidence intervals:
  - distinction between a population *parameter* and a sample *statistic* and the use of the sample statistic to estimate the population parameter
  - concept of the sample proportion $\hat{P} = \frac{X}{n}$ as a random variable whose value varies between samples, where X is a binomial random variable which is associated with the number of items that have a particular characteristic and n is the sample size
  - approximate normality of the distribution of $\hat{P}$ for large samples and, for such a situation, the mean p, (the population proportion) and standard deviation, $\sqrt{\frac{p(1-p)}{n}}$
  - simulation of random sampling, for a variety of values of p and a range of sample sizes, to illustrate the distribution of $\hat{P}$
  - determination of, from a large sample, an approximate confidence interval $\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ for a population proportion where z is the appropriate quantile for the standard normal distribution, in particular the 95% confidence interval as an example of such an interval where $z \approx 1.96$ (the term standard error may be used but is not required).

Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes as a set apply to the content from the areas of study covered in that unit.

Tick off each of these skills as you recognize what they are referring to. ✓

Outcome 1

On completion of each unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

Key knowledge

- the key features and properties of a function or relation and its graph and of families of functions and relations and their graphs
- the effect of transformations on the graphs of a function or relation
- the matrix representation of points and transformations of the plane

- the concepts of domain, maximal domain, range and asymptotic behaviour of functions
- functional relations that describe properties, symmetry and equivalence
- the concept of an inverse function, connection between domain and range of the original function and its inverse relation and the conditions for existence of an inverse function, including the form of the graph of the inverse function for specified functions
- the concept of combined functions, and the connection between domain and range of the functions involved and the domain and range of the combined functions
- the features which enable the recognition of general forms of possible models for data presented in graphical or tabular form
- index (exponent) laws, and logarithm laws
- analytical, graphical and numerical approaches to solving equations and the nature of corresponding solutions (real, exact or approximate) and the effect of domain restrictions
- features which link the graph of a function to the graph of the corresponding gradient function or its numerical values, the tangent to a curve at a given point and how the sign and magnitude of the derivative of a function can be used to describe key features of the function and its derivative function
- the sum, difference, chain, product and quotient rules for differentiation
- the properties of anti-derivatives and definite integrals
- the concept of approximation to the area under a curve using rectangles, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area
- the concepts of a random variable (discrete and continuous), bernoulli trials and probability distributions, the parameters used to define a distribution and properties of probability distributions and their graphs
- the conditions under which a bernoulli trial or a probability distribution may be selected to suitably model various situations
- the definition of sample proportion as a random variable and key features of the distribution of sample proportions
- the concept of confidence intervals for proportions, variation in confidence intervals between samples and confidence intervals for estimates.

Key skills

- identify key features and properties of the graph of a function or relation and draw the graphs of specified functions and relations, clearly identifying their key features and properties
- describe the effect of transformations on the graphs of a function or relation
- apply matrices to transformations of functions and their graphs
- find the rule of an inverse function and give its domain and range
- find the rule of a composite function and give its domain and range
- sketch by hand graphs of polynomial functions up to degree 4; simple power functions, $y = a^x$ (using key points $\left(-1, \frac{1}{a}\right)$, $(0,1)$ and $(1, a)$); $\log_e(x)$; $\log_{10}(x)$; and simple transformations of these
- apply a range of analytical, graphical and numerical processes, as appropriate, to obtain general and specific solutions (exact or approximate) to equations (including literal equations) over a given domain and be able to verify solutions to a particular equation or equations over a given domain
- solve by hand equations of the form $\sin(ax + b) = c$, $\cos(ax + b) = c$ and $\tan(ax + b) = c$ with exact value solutions over a given interval
- apply algebraic, logarithmic and circular function properties to the simplification of expressions and the solution of equations
- evaluate derivatives of basic, transformed and combined functions and apply differentiation to curve sketching and related optimisation problems

- find derivatives of polynomial functions and power functions, functions of the form $f(ax+b)$ where f is x^n, for $n \in Q$, sine, cosine; tangent, e^x, or $\log_e(x)$ and simple linear combinations of these, using pattern recognition, or by hand
- apply the product, chain and quotient rules for differentiation to simple combinations of functions by hand
- find derivatives of basic and more complicated functions and apply differentiation to curve sketching and optimisation problems
- find anti-derivatives of polynomial functions and power functions, functions of the form $f(ax+b)$ where f is x^n, for $n \in Q$, e^x, sine or cosine, and simple linear combinations of these, using pattern recognition, or by hand
- evaluate rectangular area approximations to the area under a curve, find and verify anti-derivatives of specified functions and evaluate definite integrals
- apply definite integrals to the evaluation of the area under a curve and between curves over a specified interval
- analyse a probability mass function or probability density function and the shape of its graph in terms of the defining parameters for the probability distribution and the mean and variance of the probability distribution
- calculate and interpret the probabilities of various events associated with a given probability distribution, by hand in cases where simple arithmetic computations can be carried out
- apply probability distributions to modelling and solving related problems
- simulate repeated random sampling and interpret the results, for a variety of population proportions and a range of sample sizes, to illustrate the distribution of sample proportions and variations in confidence intervals
- calculate sample proportions and confidence intervals for population proportions.

Outcome 2

On completion of each unit the student should be able to apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and skills outlined in one or more areas of study.

Key knowledge

- the key mathematical content from one or more areas of study related to a given context
- specific and general formulations of concepts used to derive results for analysis within a given context
- the role of examples, counter-examples and general cases in working mathematically
- inferences from analysis and their use to draw valid conclusions related to a given context.

Key skills

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context
- use a variety of techniques to verify results
- make inferences from analysis and use these to draw valid conclusions related to a given context
- communicate conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context.

Outcome 3

On completion of each unit the student should be able to select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

- the exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
- domain and range requirements for specification of graphs of functions and relations, when using technology
- the role of parameters in specifying general forms of functions and equations
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

Key skills

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
- select an appropriate functionality of technology in a variety of mathematical contexts, and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.