EXAMPLE 2

Consider the function f with the rule:

$$f(x) = \begin{cases} 1.5(1 - x^2) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x > 1 \text{ or } x < 0 \end{cases}$$

a Sketch the graph of f.

b Show that f is a probability density function.

c Find $\Pr(X > 0.5)$, where the random variable X has probability density function f.

EXAMPLE 1

Consider the exponential probability density function f with the rule:

$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

a Sketch the graph of f.

b Show that f is a probability density function.

c Find $\Pr(X > 1)$, where the random variable X has probability density function f.

EXAMPLE 4

A teacher travels by car to school and the journey time, *t* hours, has the probability density function given by:

$$f(t) = \begin{matrix} 10ct^2 & 0 \le t \le 0.6 \\ 9c(1-t) & 0.6 < t \le 1 \\ 0 & \text{otherwise} \end{matrix}$$

where *c* is a constant.

a. Find the value of *c*.

b. Sketch the graph of $f(t)$.

c. Calculate the median travel time.

d. Determine the probability that the travel time will be :
   i. More than 48 minutes.

   ii. Between 24 minutes and 48 minutes.

EXAMPLE 5:

The time *t*, in minutes that Jamie spends riding his bike to work is a continuous random variable with the probability density function:

$$f(t) = \begin{cases} mt(100 - t^2) & 0 < t \leq 10 \\ 0 & \text{elsewhere} \end{cases}$$

a. Calculate the value of *m*.

b. Calculate the exact mean time that Jamie takes to ride to work.

c. Calculate the standard deviation, correct to two decimal places, of the time it takes Jamie to get to work on his bike.

d. Calculate the interval within which lie the middle 50% of times it takes for Jamie to get to work, correct to two decimal places.

EXAMPLE 3

The queuing time in minutes, X, of a customer at a post office is modelled by the probability density function

$$f(x) = \begin{cases} kx(81 - x^2) & 0 \leqslant x \leqslant 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{4}{6561}$.

(3)

Using integration, find

(b) the mean queuing time of a customer,

(4)

(c) the probability that a customer will queue for more than 5 minutes.