SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The point $P(4, -3)$ lies on the graph of a function f. The graph of f is translated four units vertically up and then reflected in the y-axis.

The coordinates of the final image of P are

A. $(-4, 1)$
B. $(-4, 3)$
C. $(0, -3)$
D. $(4, -6)$
E. $(-4, -1)$

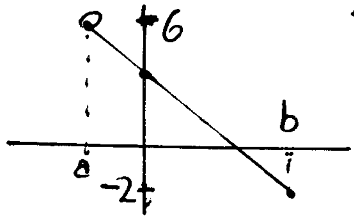
$(4, -3) \rightarrow (4, 1) \rightarrow (-4, 1)$

Question 2

The linear function $f: D \rightarrow R, f(x) = 4 - x$ has range $[-2, 6)$.

The domain D of the function is

A. $[-2, 6)$
B. $(-2, 2]$
C. R
D. $(-2, 6]$
E. $[-6, 2]$



$4 - a = 6$
$\therefore a = -2$

$4 - b = -2$
$\therefore b = 6$

$\therefore D = (-2, 6]$

Question 3

The area of the region enclosed by the graph of $y = x(x+2)(x-4)$ and the x-axis is

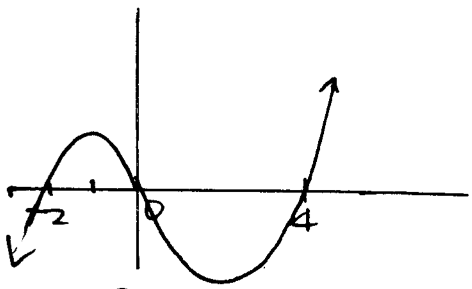
A. $\frac{128}{3}$

B. $\frac{20}{3}$

C. $\frac{236}{3}$

D. $\frac{148}{3}$

E. 36



Define f(x) = x(x+2)(x-4) and then evaluate the integrals remembering to subtract them as the second one is negative.

$$\int_{-2}^{0} x(x+2)(x-4)\,dx - \int_{0}^{4} x(x+2)(x-4)\,dx$$

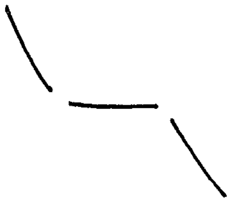
$$= \frac{148}{3}$$

Question 4

Let f be a function with domain R such that $f'(5)=0$ and $f'(x)<0$ when $x \neq 5$.

At $x=5$, the graph of f has a

A. local minimum.
B. local maximum.
C. gradient of 5.
D. gradient of –5.
E. stationary point of inflection.



Question 5

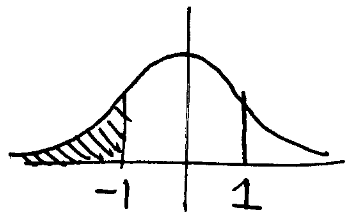
The random variable X has a normal distribution with mean 12 and standard deviation 0.5.

If Z has the standard normal distribution, then the probability that X is less than 11.5 is equal to

A. $\Pr(Z > -1)$
B. $\Pr(Z < -0.5)$
C. $\Pr(Z > 1)$
D. $\Pr(Z \geq 0.5)$
E. $\Pr(Z < 1)$

$X \overset{d}{=} N(\mu = 12, \sigma = 0.5)$

$Pr(X < 11.5) = Pr\left(Z < \frac{11.5-12}{0.5}\right) = Pr(Z < -1)$

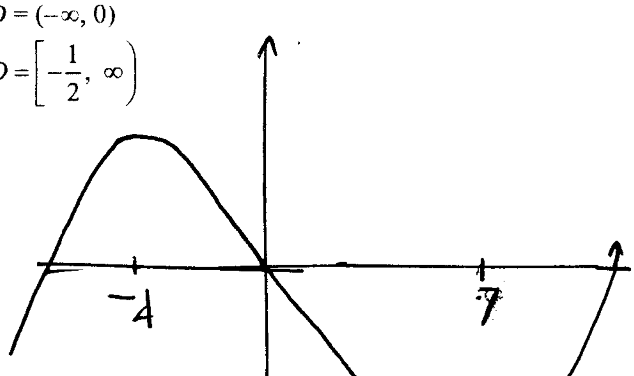


$Pr(Z<-1) = Pr(Z>1)$

Question 6

The function $f : D \to R$ with rule $f(x) = 2x^3 - 9x^2 - 168x$ will have an inverse function for

A. $D = R$
B. $D = (7, \infty)$
C. $D = (-4, 8)$
D. $D = (-\infty, 0)$
E. $D = \left[-\frac{1}{2}, \infty\right)$



For an inverse to exist, f must be a one-to-one function. Thus the domain we require must contain no turning points

$\therefore (7, \infty)$ is an appropriate domain

Q7 is no longer on the curriculum.

Question 8

If $\int_1^4 f(x)\,dx = 6$, then $\int_1^4 (5 - 2f(x))\,dx$ is equal to

A. 3
B. 4
C. 5
D. 6
E. 16

$$\int_1^4 (5 - 2f(x))\,dx$$

$$= \int_1^4 5\,dx - 2\int_1^4 f(x)\,dx$$

$$= [5x]_1^4 - 2 \times 6$$

$$= 15 - 12$$

$$= 3$$

Question 9

The inverse of the function $f: R^+ \to R,\ f(x) = \dfrac{1}{\sqrt{x}} + 4$ is

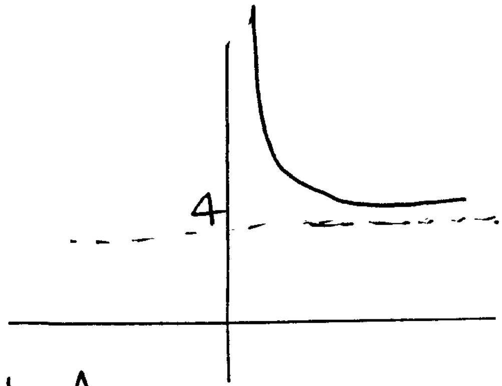
A. $f^{-1}: (4, \infty) \to R \qquad f^{-1}(x) = \dfrac{1}{(x-4)^2}$

B. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = \dfrac{1}{x^2} + 4$

C. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = (x+4)^2$

D. $f^{-1}: (-4, \infty) \to R \qquad f^{-1}(x) = \dfrac{1}{(x+4)^2}$

E. $f^{-1}: (-\infty, 4) \to R \qquad f^{-1}(x) = \dfrac{1}{(x-4)^2}$



Only A has the correct domain

dom(f)	ran(f)
$(0,\infty)$	$(4,\infty)$
dom(f^{-1})	ran(f^{-1})
$(4,\infty)$	$(0,\infty)$

Question 10

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

A: $f(2x) = 2(2x) = 4x \neq x$

B: $f(x^2) = (x^2)^2 = x^4 \neq x$

C: $f(2\sqrt{x}) = \sqrt{2\sqrt{x}} \neq x$

D: $f(x-2) = x - 4 \neq x$

E: $f(2-x) = 2-(2-x)$
$= x$

Question 11

A bag contains five red marbles and four blue marbles. Two marbles are drawn from the bag, without replacement, and the results are recorded.

The probability that the marbles are different colours is

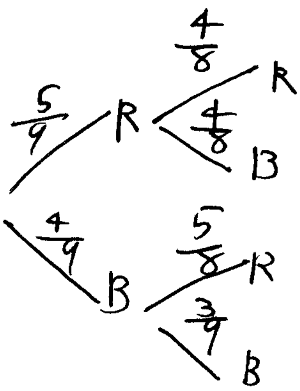
A. $\dfrac{20}{81}$

B. $\dfrac{5}{18}$

C. $\dfrac{4}{9}$

D. $\dfrac{40}{81}$

E. $\dfrac{5}{9}$



$\Pr(RB) + \Pr(BR)$

$= \dfrac{5}{9} \times \dfrac{1}{2} + \dfrac{4}{9} \times \dfrac{5}{8}$

$= \dfrac{5}{18} + \dfrac{20}{72}$

$= \dfrac{5}{18} + \dfrac{5}{18} = \dfrac{10}{18} = \dfrac{5}{9}$

Question 12

The transformation $T: R^2 \to R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -x+1 \\ 2y-2 \end{bmatrix}$$

maps the line with equation $x - 2y = 3$ onto the line with equation

A. $x + y = 0$

B. $x + 4y = 0$

C. $-x - y = 4$

D. $x + 4y = -6$

E. $x - 2y = 1$

$\therefore x' = -x+1 \quad \therefore x = 1-x'$

$y' = 2y-2 \quad \therefore y = \frac{y'+2}{2}$

$\therefore 1-x' - \frac{2(y'+2)}{2} = 3$

$1-x' - y' - 2 = 3$

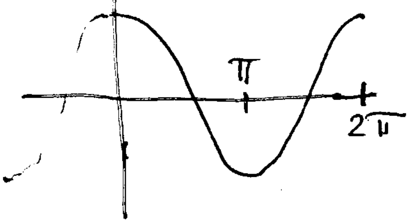
$\therefore -x' - y' = 4$

Question 13

The domain of the function h, where $h(x) = \cos(\log_a(x))$ and a is a real number greater than 1, is chosen so that h is a one-to-one function.

Which one of the following could be the domain?

$y = \cos x$ is one-to one on domain $[0, \pi]$



A. $\left(a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right)$

B. $(0, \pi)$

C. $\left[1, a^{\frac{\pi}{2}}\right]$

D. $\left[a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right)$

E. $\left[a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right]$

$\log_a x = 0$ if $x = 1$

If $x = a^{\pi/2}$ then $\cos\left(\log_a a^{\frac{\pi}{2}}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$\therefore h(x)$ is one-one one for $1 \leq x \leq a^{\frac{\pi}{2}}$

Question 14

If X is a random variable such that $\Pr(X > 5) = a$ and $\Pr(X > 8) = b$, then $\Pr(X < 5 | X < 8)$ is

A. $\frac{a}{b}$

B. $\frac{a-b}{1-b}$

C. $\frac{1-b}{1-a}$

D. $\frac{ab}{1-b}$

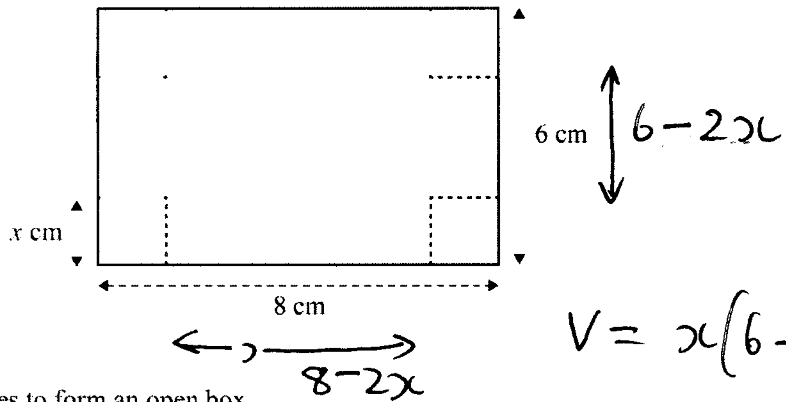
E. $\frac{a-1}{b-1}$

$$\Pr(X<5 | X<8) = \frac{\Pr(X<5 \cap X<8)}{\Pr(X<8)}$$

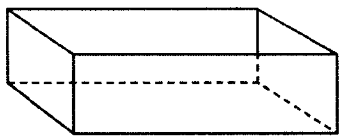
$$= \frac{\Pr(X<5)}{\Pr(X<8)} = \frac{1-a}{1-b} = \frac{a-1}{b-1}$$

Question 15

Zoe has a rectangular piece of cardboard that is 8 cm long and 6 cm wide. Zoe cuts squares of side length x centimetres from each of the corners of the cardboard, as shown in the diagram below.



Zoe turns up the sides to form an open box.



$$V = x(6-2x)(8-2x)$$
$$= 4x(3-x)(4-x)$$
$$= 4x(12-7x+x^2)$$
$$= 4(12x-7x^2+x^3)$$
$$V'(x) = 0$$
$$\therefore 12 - 14x + 3x^2 = 0$$
Solving: $x = 1.13 \ (x < 3)$

The value of x for which the volume of the box is a maximum is closest to

A. 0.8
B. 1.1
C. 1.6
D. 2.0
E. 3.6

Question 16

The continuous random variable X, with probability density function $p(x)$, has mean 2 and variance 5.

The value of $\int_{-\infty}^{\infty} x^2 p(x)dx$ is

A. 1
B. 7
C. 9
D. 21
E. 29

$$\text{Var}(x) = \int_{-\infty}^{\infty} x^2 p(x)dx - \mu^2$$
$$\therefore 5 = \int_{-\infty}^{\infty} x^2 p(x)dx - 2^2$$
$$\therefore \int_{-\infty}^{\infty} x^2 p(x)dx = 9$$

Question 17

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for

A. $a = 3$
B. $a = -3$
C. both $a = 3$ and $a = -3$
D. $a \in R\backslash\{3\}$
E. $a \in R\backslash[-3, 3]$

$$\begin{bmatrix} a & -3 \\ 3 & -a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8-a \end{bmatrix}$$

$\det(A) = 0$
$\therefore -a^2 + 9 = 0$
$\therefore a = \pm 3$

If $a = 3$:
$\left.\begin{matrix} 3x - 3y = 5 \\ 3x - 3y = 5 \end{matrix}\right]$ same equation

If $a = -3$:
$\left.\begin{matrix} -3x - 3y = 5 \\ 3x + 3y = 11 \end{matrix}\right]$ parallel lines ∴ no solution

Question 18

The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for

A. $k = 6$

B. $k > 6$ or $k < -2$

C. $-2 \le k \le 6$

D. $6 - 2\sqrt{3} \le k \le 6 + 2\sqrt{3}$

E. $k = -2$

$x^2 + 2x = kx - 4$

$x^2 + (2-k)x + 4 = 0$

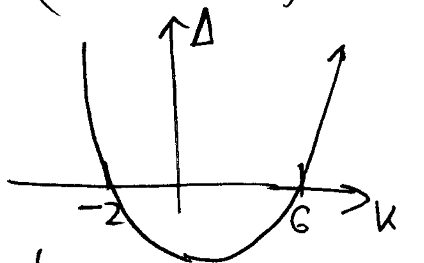
$\Delta > 0 \therefore (2-k)^2 - 4 \times 4 \times 1 > 0$

$\therefore (2-k)^2 - 16 > 0$

$(2-k-4)(2-k+4) > 0$

$\therefore (-k-2)(6-k) > 0$

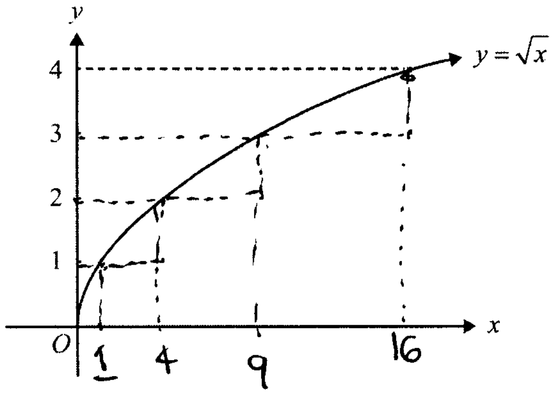
$(k+2)(k-6) > 0$



$\{\therefore k : k < -2 \cup k : k > 6\}$

Question 19

Jake and Anita are calculating the area between the graph of $y = \sqrt{x}$ and the y-axis between $y = 0$ and $y = 4$. Jake uses a partitioning, shown in the diagram below, while Anita uses a definite integral to find the exact area.



The difference between the results obtained by Jake and Anita is

A. 0

B. $\frac{22}{3}$

C. $\frac{26}{3}$

D. 14

E. 35

Jake

Area $= 1 \times 1 + 4 \times 1 + 9 \times 1 + 16 \times 1$

$= 30$

$\therefore$ Difference

$= 30 - \frac{64}{3}$

$= \frac{26}{3}$

Anita

Rectangle Area − Integral

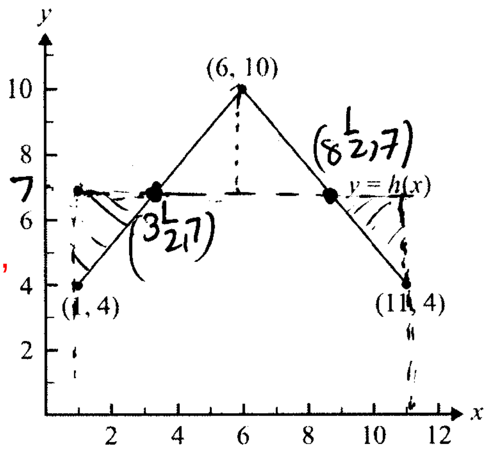
$64 - \int_0^{16} \sqrt{x}\, dx$

$= 64 - \left[\frac{2}{3} x^{3/2}\right]_0^{16}$

$= 64 - \frac{2}{3} \times 64 = \frac{64}{3}$

Question 20

The graph of a function, h, is shown below.



When calculating the average value of a function over an interval, the area above the rectangle is equal to the area missing from it.

Area of two shaded triangles must equal the area of two triangles at the top

By the geometry, the height of the rectangle must be 7.

$$\therefore \frac{1}{11-1}\int_1^{11} f(x)\,dx = 7$$

The average value of h is

A. 4

B. 5

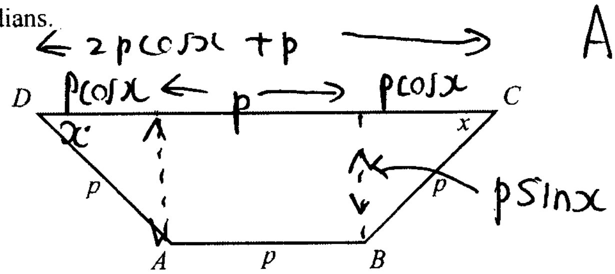
C. 6

D. 7

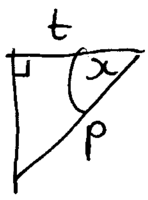
E. 10

Question 21

The trapezium $ABCD$ is shown below. The sides AB, BC and DA are of equal length, p. The size of the acute angle BCD is x radians.



A =



$\cos x = \frac{t}{p}$

$\therefore t = p\cos x$

The area of the trapezium is a maximum when the value of x is

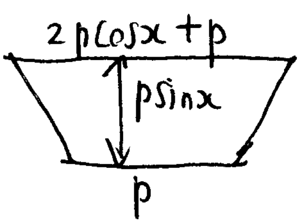
A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

E. $\frac{5\pi}{12}$



$A = \frac{1}{2}h(a+b)$

$\therefore A = \frac{1}{2}p\sin x(2p\cos x + p + p)$

$= p\sin x(p\cos x + p)$

$A(x) = p^2\sin x(1+\cos x)$

For a maximum,

$A'(x) = 0$ for $0 < x < \frac{\pi}{2}$

$\therefore x = \frac{\pi}{3}$

Define A(x) on CAS and dA(x), the derivative. Let dA(x) equal zero and solve

Note domain!

Question 22

John and Rebecca are playing darts. The result of each of their throws is independent of the result of any other throw. The probability that John hits the bullseye with a single throw is $\frac{1}{4}$. The probability that Rebecca hits the bullseye with a single throw is $\frac{1}{2}$. John has four throws and Rebecca has two throws.

The ratio of the probability of Rebecca hitting the bullseye at least once to the probability of John hitting the bullseye at least once is

A. 1:1
B. 32:27
C. 64:85
D. 2:1
E. 192:175

For John: X = no. of hits

$X \stackrel{d}{=} Bi(n=4, p=\frac{1}{4})$

$Pr(X \geq 1) = 1 - Pr(X=0) = 1 - \left(\frac{3}{4}\right)^4$

$= 1 - \frac{81}{256}$

$= \frac{175}{256}$

Rebecca: $X \stackrel{d}{=} Bi(n=2, p=\frac{1}{2})$

$1 - Pr(X=0) = 1 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4}$

$\frac{3}{4} : \frac{175}{256}$

$= 3 : \frac{175}{64}$

$= 192 : 175$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

The population of wombats in a particular location varies according to the rule $n(t) = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$, where n is the number of wombats and t is the number of months after 1 March 2013.

a. Find the period and amplitude of the function n. 2 marks

Period $= \dfrac{2\pi}{\frac{\pi}{3}} = 6$ months

Amplitude $= 400$

b. Find the maximum and minimum populations of wombats in this location. 2 marks

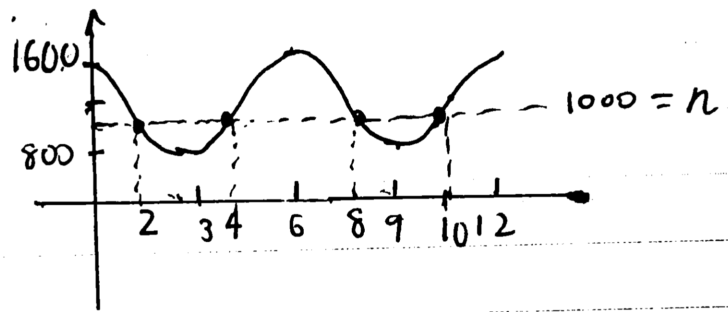
↑ 1600
1200
↓ 800

Max $= 1600$
Min $= 800$

c. Find $n(10)$. 1 mark

$n(10) = 1200 + 400\cos\left(\frac{10\pi}{3}\right)$
$= 1200 - 200 = 1{,}000$

d. Over the 12 months from 1 March 2013, find the fraction of time when the population of wombats in this location was less than $n(10)$. 2 marks

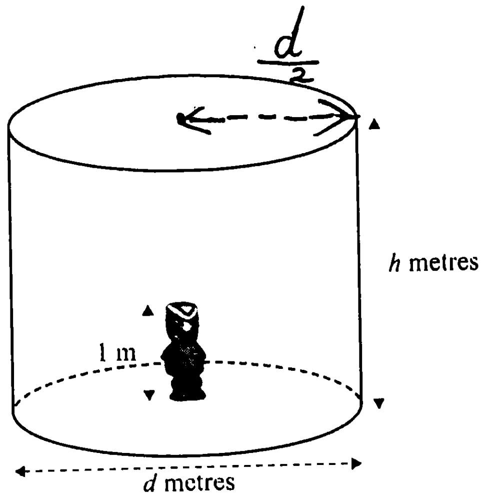


Total no. of hours $n < 1000$:
$(4-2) + (10-8) = 4$
$\therefore$ Fraction $= \frac{4}{12} = \frac{1}{3}$

Question 2 (13 marks)

On 1 January 2010, Tasmania Jones was walking through an ice-covered region of Greenland when he found a large ice cylinder that was made a thousand years ago by the Vikings.

A statue was inside the ice cylinder. The statue was 1 m tall and its base was at the centre of the base of the cylinder.



The cylinder had a height of h metres and a diameter of d metres. Tasmania Jones found that the volume of the cylinder was 216 m^3. At that time, 1 January 2010, the cylinder had not changed in a thousand years. It was exactly as it was when the Vikings made it.

a. Write an expression for h in terms of d. 2 marks

$$V = \pi r^2 h$$

$$\therefore V = \pi\left(\frac{d}{2}\right)^2 h$$

$$\therefore V = \frac{\pi d^2 h}{4}$$

$$\therefore 216 = \frac{\pi d^2 h}{4}$$

$$\therefore h = \frac{864}{\pi d^2}$$

b. Show that the surface area of the cylinder excluding the base, S square metres, is given by the rule $S = \frac{\pi d^2}{4} + \frac{864}{d}$. 1 mark

$$S = 2\pi r h + \pi r^2$$

$$\therefore S = 2\pi\left(\frac{d}{2}\right)\frac{864}{\pi d^2} + \pi\left(\frac{d}{2}\right)^2$$

$$\therefore S = \frac{864}{d} + \frac{\pi d^2}{4}$$

Tasmania found that the Vikings made the cylinder so that S is a minimum.

c. Find the value of d for which S is a minimum and find this minimum value of S. 2 marks

$$S(d) = \frac{\pi d^2}{4} + \frac{864}{d}$$

For a minimum, $S'(d) = 0$

$$\therefore \frac{\pi d}{2} - \frac{864}{d^2} = 0$$

$$d^3 = \frac{1728}{\pi}$$

$$d = \sqrt[3]{\frac{1728}{\pi}} = 12\left(\frac{1}{\pi}\right)^{1/3}$$

$$S_{min} = S\left(\frac{12}{\pi^{1/3}}\right)$$

$$= 108\pi^{1/3}$$

d. Find the value of h when S is a minimum. 1 mark

$$h = \frac{6}{\pi^{1/3}}$$

CAS TIP:

$$\left(\text{Define } h(d) = \frac{864}{\pi d^2}\right.$$

$$h\left(\frac{12}{\pi^{1/3}}\right) = \frac{6}{\pi^{1/3}}$$

Question 4 (14 marks)

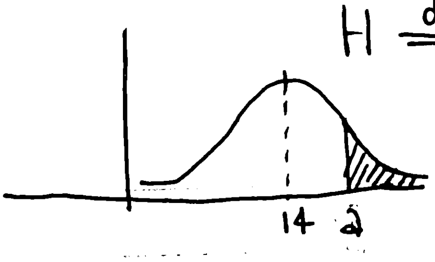
Patricia is a gardener and she owns a garden nursery. She grows and sells basil plants and coriander plants.

The heights, in centimetres, of the basil plants that Patricia is selling are distributed normally with a mean of 14 cm and a standard deviation of 4 cm. There are 2000 basil plants in the nursery.

a. Patricia classifies the tallest 10 per cent of her basil plants as **super**.

What is the minimum height of a super basil plant, correct to the nearest millimetre? 1 mark

$$H \overset{d}{=} N(\mu = 14, \sigma = 4)$$



$$\boxed{a = \text{invNorm}(14, 4, 0.9)}$$

CAS only (You would not write this as part of your working)

$a = 19.126$ cm ≈ 191 mm

Patricia decides that some of her basil plants are not growing quickly enough, so she plans to move them to a special greenhouse. She will move the basil plants that are less than 9 cm in height.

b. How many basil plants will Patricia move to the greenhouse, correct to the nearest whole number? 2 marks

$\Pr(H < 9) = 0.10565$

0.10565×2000

$= 211$

$\therefore$ 211 plants

The heights of the coriander plants, x centimetres, follow the probability density function $h(x)$, where

$$h(x)=\begin{cases}\dfrac{\pi}{100}\sin\left(\dfrac{\pi x}{50}\right) & 0<x<50\\ 0 & \text{otherwise}\end{cases}$$

c. State the mean height of the coriander plants. 1 mark

$$E(X)=\int_0^{50}\frac{\pi x}{100}\sin\left(\frac{\pi x}{50}\right)dx$$

$$=25$$

Patricia thinks that the smallest 15 per cent of her coriander plants should be given a new type of plant food.

d. Find the maximum height, correct to the nearest millimetre, of a coriander plant if it is to be given the new type of plant food. 2 marks

$\Pr(X\leq b)=0.15$ $\quad\int_0^{b}\frac{\pi}{100}\sin\left(\frac{\pi x}{50}\right)dx=0.15$, where $0<b<50$

Solving: $b=12.6592$

$\therefore b\simeq 127$ mm

Patricia also grows and sells tomato plants that she classifies as either **tall** or **regular**. She finds that 20 per cent of her tomato plants are tall.

A customer, Jack, selects n tomato plants at random.

e. Let q be the probability that at least one of Jack's n tomato plants is tall.

Find the minimum value of n so that q is greater than 0.95. 2 marks

Let X = no. of tomatos that are tall

$X\overset{d}{=}Bi(n, p=0.2)$

$\Pr(X\geq 1)\geq 0.95$

$1-\Pr(X=0)\geq 0.95$

$\therefore \Pr(X=0)<0.05$

First, solve: $\Pr(X=0)=0.05$

$\binom{n}{0}(0.2)^0(0.8)^n=0.05$

$n=13.425$

$\therefore n=14$

Question 5 (13 marks)

Let $f: R \to R, f(x) = (x-3)(x-1)(x^2+3)$ and $g: R \to R, g(x) = x^4 - 8x$.

a. Express $x^4 - 8x$ in the form $x(x-a)((x+b)^2+c)$. 2 marks

$$x^4 - 8x = x(x^3 - 8)$$
$$= x(x-2)(x^2 + 2x + 4)$$
$$= x(x+2)(x^2+2x+1^2-1^2+4) = x(x+2)((x+1)^2+3)$$

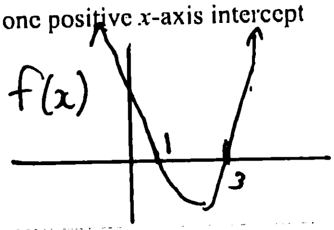
b. Describe the translation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. 1 mark

$$f(x) = (x-3)(x-1)(x^2+3)$$
$$f(x+1) = (x+1-3)(x+1-1)((x+1)^2+3)$$
$$= (x-2)(x)((x+1)^2+3) = g(x)$$

$\therefore$ f is translated 1 unit to the left to obtain the graph of $g(x)$.

c. Find the values of d such that the graph of $y = f(x+d)$ has

i. one positive x-axis intercept 1 mark



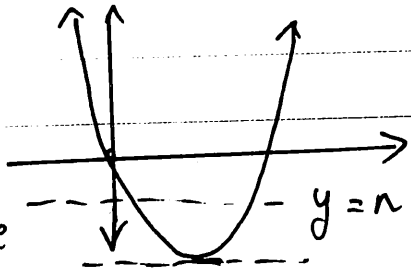
For 1 positive x-intercept
$1 \leq d < 3$

ii. two positive x-axis intercepts. 1 mark

For 2 positive x-intercepts:
$d < 1$

d. Find the value of n for which the equation $g(x) = n$ has one solution. 1 mark

$$g(x) = x^4 - 8x$$



For one solution,
n is equal to the y co-ordinate of the turning point.

$$g'(x) = 4x^3 - 8 = 0$$
$$\therefore x^3 = 2 \quad \therefore x = 2^{\frac{1}{3}}$$
$$g(2^{1/3}) = -6 \times 2^{\frac{1}{3}} \qquad \therefore n = -6 \times 2^{\frac{1}{3}}$$

SECTION 2 – Question 5 – continued
TURN OVER

e. At the point $(u, g(u))$, the gradient of $y = g(x)$ is m and at the point $(v, g(v))$, the gradient is $-m$, where m is a positive real number.

i. Find the value of $u^3 + v^3$. 2 marks

$g'(x) = 4x^3 - 8$

$g'(u) = m \quad \therefore 4u^3 - 8 = m$

$g'(v) = -m \quad \therefore 4v^3 - 8 = -m$

$\therefore 4u^3 - 8 + 4v^3 - 8 = 0$

$\therefore 4u^3 + 4v^3 = 16$

$\therefore u^3 + v^3 = 4$

ii. Find u and v if $u + v = 1$. 1 mark

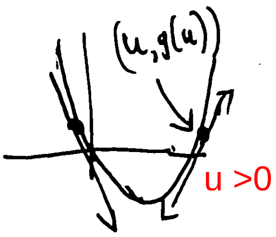
$u^3 + v^3 = 4$

$u + v = 1$

Solving :

$u = \dfrac{\sqrt{5}+1}{2}, \quad v = \dfrac{1-\sqrt{5}}{2}$

(since u is positive and v is negative).



f. **i.** Find the equation of the tangent to the graph of $y = g(x)$ at the point $(p, g(p))$. 1 mark

$g'(p) = 4p^3 - 8$

$\therefore y - g(p) = (4p^3 - 8)(x - p)$ is tangent equation.

ii. Find the equations of the tangents to the graph of $y = g(x)$ that pass through the point with coordinates $\left(\frac{3}{2}, -12\right)$. 3 marks

$g(p) = p^4 - 8p$

$\therefore y - (p^4 - 8p) = (4p^3 - 8)(x - p)$

Since tangent goes through $\left(\frac{3}{2}, -12\right)$, we get:

$-12 - p^4 + 8p = (4p^3 - 8)\left(\frac{3}{2} - p\right)$

Solving : $p = 0, 2$

If $p = 0$: Equation is : $y = -8x$

If $p = 2$: Equation is : $y - (16 - 16) = 24(x - 2)$

$\therefore y = 24x - 48$

END OF QUESTION AND ANSWER BOOK

$\therefore$ Tangents are : $y = -8x$

$y = 24x - 48$