Question 2 (18 marks)

Rebecca's Robotics manufactures three types of components for robots: sensors, motors and controllers. The manufacturing processes for each type of component are independent.

It is known that 8% of all of the sensors manufactured are defective.

a. A random sample of five sensors is selected.

Find, correct to four decimal places, the probability that

i. exactly two of these selected sensors are defective — 2 marks

Let $X =$ # defective sensors

$X \sim Bi(n=5, p=0.08) \therefore \Pr(X=2) \approx 0.0498$

ii. exactly two of these selected sensors are defective, given that at most two sensors in the sample are defective. — 2 marks

$\Pr(X=2 \mid X \leq 2) = \dfrac{\Pr(X=2)}{\Pr(X \leq 2)}$

$= \dfrac{0.0498\ldots}{0.99547\ldots} \approx 0.0501$

b. A random sample of 50 sensors is selected and it is found that the proportion of defective sensors in this sample is 0.08

Determine an approximate 90% confidence interval for the proportion of defective sensors, correct to four decimal places. — 2 marks

$\hat{p} = 0.08, \; n = 50$

$\left(\hat{p} - 1.645\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \; \hat{p} + 1.645\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right) \rightarrow (0.0169, 0.1411)$

A hole is drilled into each motor. The depth of the hole is normally distributed with a mean of 20 mm and a standard deviation of 0.3 mm.

c. What is the probability that, for a randomly selected motor, the depth of the hole is greater than 20.6 mm? Give your answer correct to four decimal places. — 1 mark

$H \sim N(\mu = 20, \sigma = 0.3)$

$\Pr(H > 20.6) = 0.0228$

The depth of the hole drilled into a motor must be within 0.5 mm of the mean, otherwise the motor is defective.

d. What is the probability that a motor is defective, correct to four decimal places? 2 marks

$\Pr(19.5 < H < 20.5)$

$\Pr(\text{motor is defective}) = 1 - 0.9044 = 0.0956$

e. Rebecca delivers an order for five sensors and five motors.

What is the probability that the order contains exactly two defective components? Give your answer correct to three decimal places. 3 marks

2 defective motors AND 0 defective sensors + 1 defective motor AND 1 defective sensor + 2 defective sensors AND 0 defective motors

$X \sim Bi(n=5, p=0.08)$ X = no. of defective sensors

$Y \sim Bi(n=5, p=0.0956)$ Y = no. of defective motors

$\Pr(X=2) \times \Pr(Y=0) + \Pr(X=1) \times \Pr(Y=1) + \Pr(X=0) \times \Pr(Y=2)$

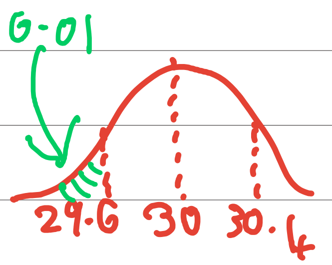
$= 0.049836 \times 0.605066 + 0.2865572 \times 0.3197939 + 0.65908152 \times 0.0676079$

≈ 0.166

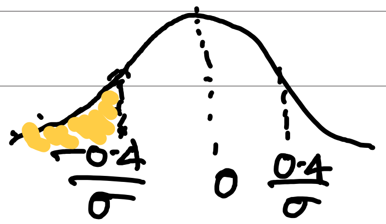
f. A knob is attached to each controller. The height of a knob is normally distributed with a mean of 30 mm. If the knob on a controller has a height greater than 30.4 mm or less than 29.6 mm, then the controller is defective.

Rebecca wants to ensure that less than 2% of all controllers manufactured are defective.

What is the maximum standard deviation of the height of a knob, in millimetres, that can be attached to a controller so that less than 2% of controllers are defective? Give your answer correct to two decimal places. 2 marks



$Z = \frac{H-30}{\sigma}$



$Z_1 = \frac{29.6-30}{\sigma}$, $Z_2 = \frac{30.4-30}{\sigma}$

$\frac{-0.4}{\sigma} = -2.327$

$\sigma = 0.17$

The weight, w, in grams, of controllers is modelled by the following probability density function.

$$C(w) = \begin{cases} \dfrac{3}{640000}(330-w)^2(w-290) & 290 \le w \le 330 \\ 0 & \text{elsewhere} \end{cases}$$

g. Determine the mean weight, in grams, of the controllers. 2 marks

$$E(W) = \int_{240}^{330} w \cdot \frac{3}{640000}(330-w)^2(w-290)\,dw$$

$$= 306$$

h. Determine the probability that a randomly selected controller weighs less than the mean weight of the controllers. Give your answer correct to four decimal places. 2 marks

$$\Pr(W < 306) = \int_{290}^{306} \frac{3}{640000}(330-w)^2(w-290)\,dw$$

$$= 0.5248$$