Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an exact answer is required to a question.
- Where an exact answer is required to a question, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

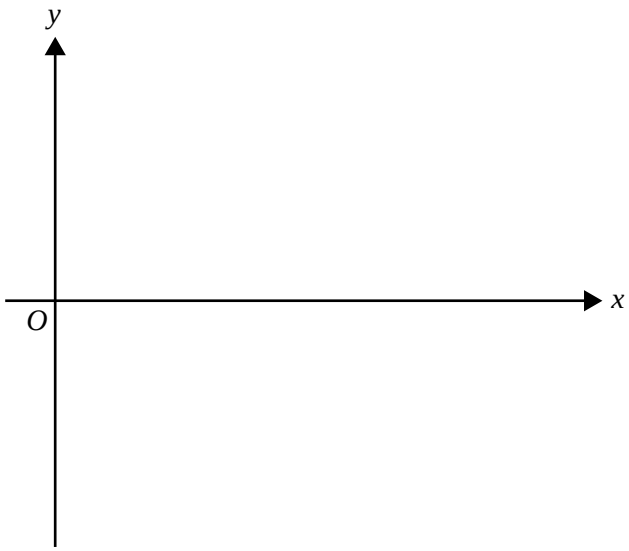
A well-designed computer screen display aims to make it quick and easy for a user to do tasks such as clicking on a screen button. Fitts' Law models the way in which the time taken to move to, and click on, a screen button depends on the distance the mouse is moved and the width of the screen button.

According to Fitts' Law, for a fixed distance travelled by the mouse, the time taken, in seconds, is given by $a - b \log_e(x)$, $0 < x \leq 5$, where x cm is the button width and a and b are positive constants for a particular user.

a. Minnie discovers that, for her, $a = 1.1$ and $b = 0.5$.

i. Let $f\colon (0, 5] \rightarrow R$, $f(x) = 1.1 - 0.5 \log_e(x)$.

Sketch the graph of $y = f(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates.



3 marks

ii. Explain why f^{-1}, the inverse function of f, exists.

1 mark

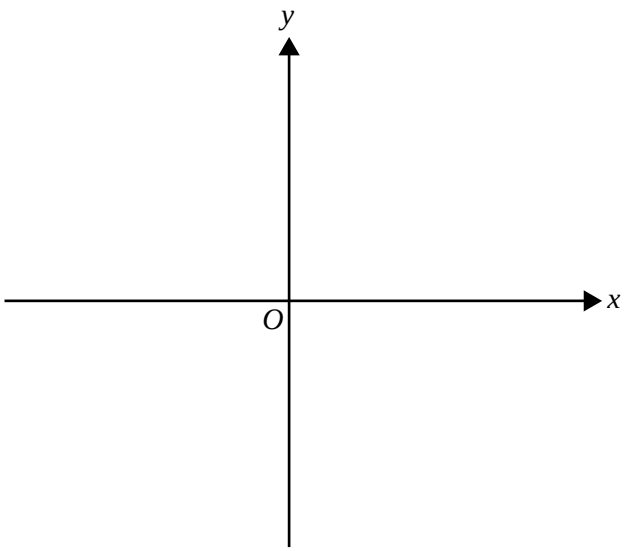
iii. Find $f^{-1}(x)$, the rule for f^{-1}.

2 marks

iv. State the domain of f^{-1}.

1 mark

v. Sketch the graph of $y = f^{-1}(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates.



2 marks

b. Mickey decides to find the values of a and b for his use. He finds that when x is 1, his time is 0.5 seconds, and when x is 1.5, his time is 0.3 seconds.

Find the exact values of a and b for Mickey.

2 marks

c. Show that, when the button width is halved, the time taken by Minnie (for whom $a = 1.1$ and $b = 0.5$) is increased by $\log_e \sqrt{2}$ seconds.

3 marks

Total 14 marks