

STUDENT NUMBER Letter

FURTHER MATHEMATICS

Written examination 2

Day Date

Reading time: *.*.* to *.*.* (15 minutes)

Writing time: *.*.* to *.*.* (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	9	9	36
Section B – Modules	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 30 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

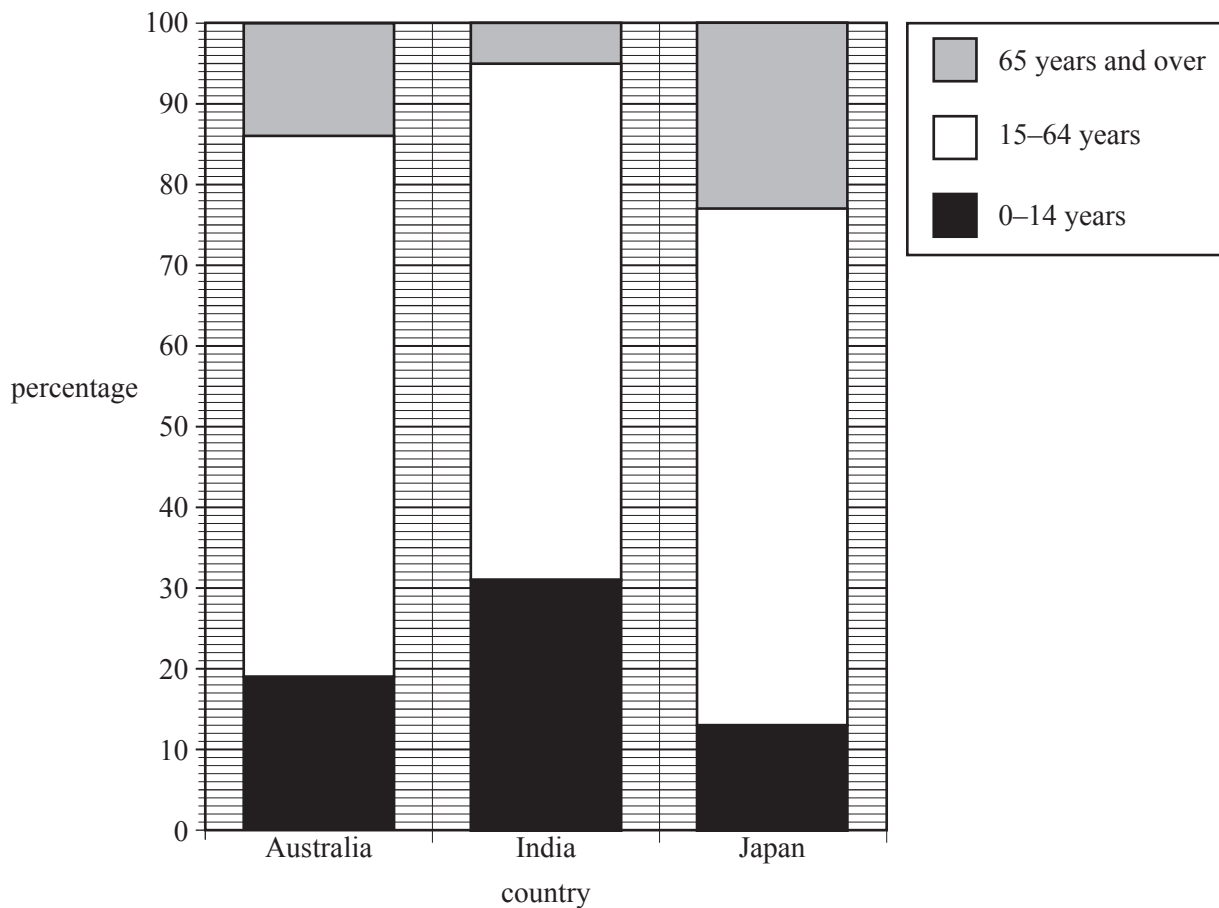
Instructions for Section A

Answer **all** questions in the spaces provided. Write using blue or black pen.
 You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.
 In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.
 Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (3 marks)

The segmented bar chart below shows the age distribution of people in three countries, Australia, India and Japan, for the year 2010.



Source: Australian Bureau of Statistics, 3201.0 – *Population by Age and Sex, Australian States and Territories*, June 2010

DO NOT WRITE IN THIS AREA

- a. Write down the percentage of people in Australia who were aged 0–14 years in 2010. 1 mark

- b. In 2010, the population of Japan was 128 000 000.
How many people in Japan were aged 65 years and over in 2010? 1 mark

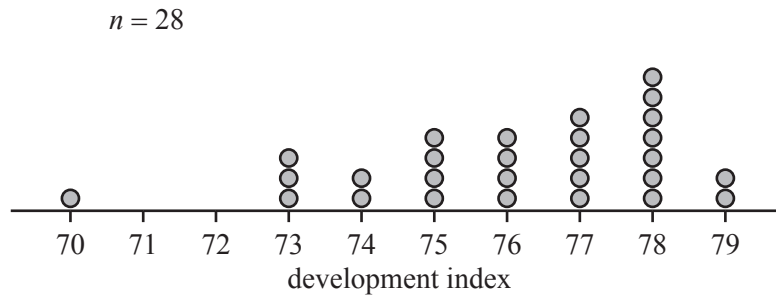
- c. From the graph on page 2, it appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live.
Explain why, quoting appropriate percentages to support your explanation. 1 mark

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Question 2 (3 marks)

The development index for a country is a whole number between 0 and 100.

The dot plot below displays the values of the development indices for 28 countries.



- a. Using the information in the dot plot, determine each of the following. 1 mark

The mode

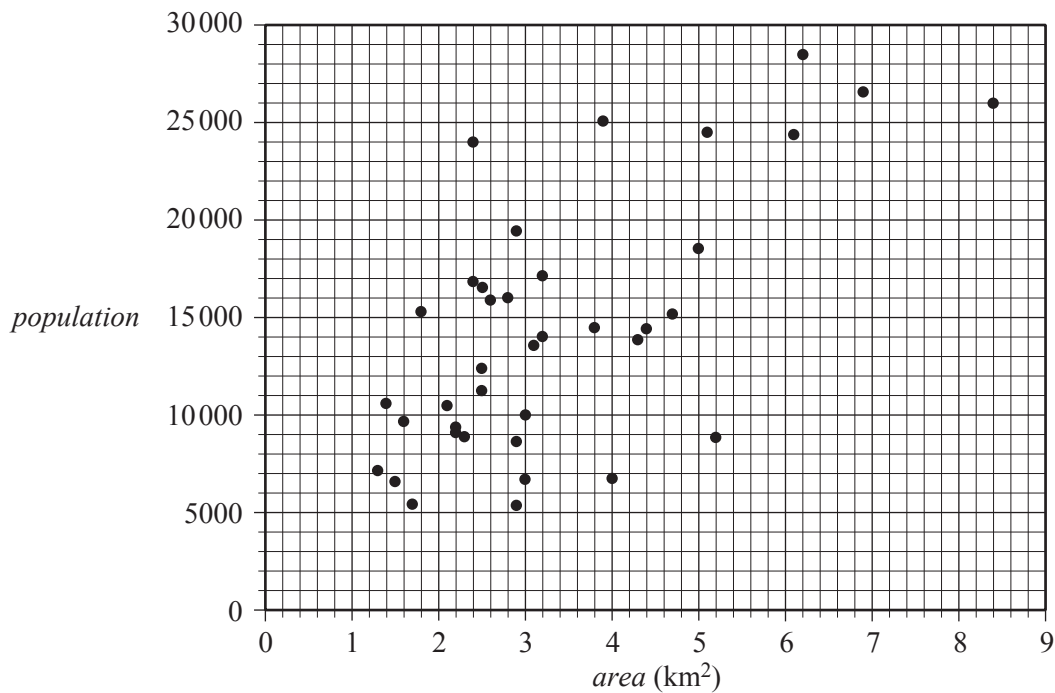
The range

- b. Write down an appropriate calculation and use it to explain why the country with a development index of 70 is an outlier for this group of countries. 2 marks

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Question 3 (6 marks)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



The equation of the least squares regression line for the data in the scatterplot is

$$\text{population} = 5330 + 2680 \times \text{area}$$

- a. Write down the response variable. 1 mark
- _____
- b. Draw the least squares regression line on the **scatterplot above**. 1 mark
- (Answer on the scatterplot above.)
- c. Interpret the slope of this least squares regression line in terms of the variables *area* and *population*. 2 marks

d. Wiston is an inner suburb. It has an area of 4 km^2 and a population of 6690.

The correlation coefficient, r , is equal to 0.668

- i. Calculate the residual when the least squares regression line is used to predict the population of Wiston from its area.

1 mark

- ii. What percentage of the variation in the population of the suburbs is explained by the variation in area?

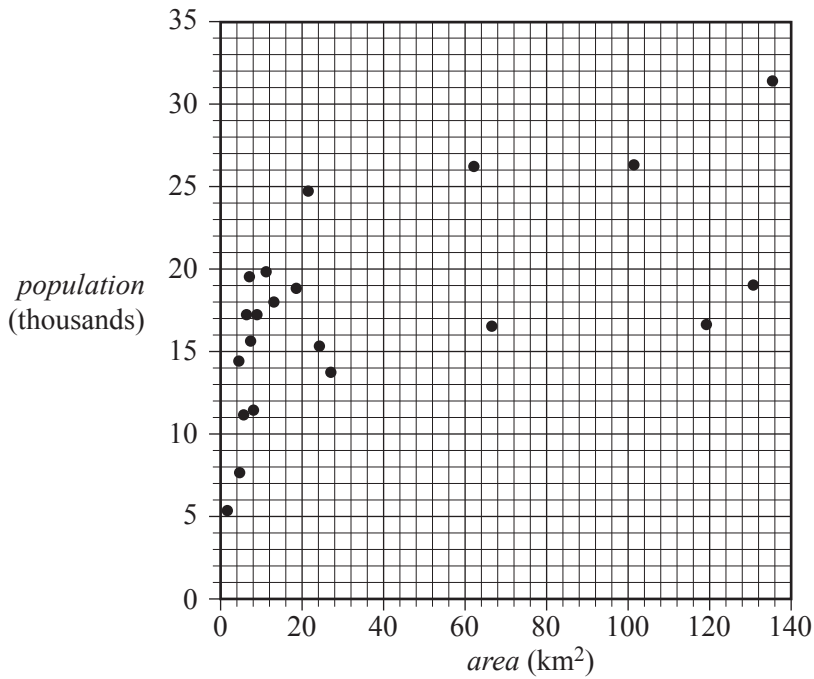
Round your answer to one decimal place.

1 mark

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Question 4 (3 marks)

The scatterplot and table below show the *population*, in thousands, and the *area*, in square kilometres, for a sample of 21 outer suburbs of the same city.



<i>Area</i> (km ²)	<i>Population</i> (thousands)
1.6	5.2
4.4	14.3
4.6	7.5
5.6	11.0
6.3	17.1
7.0	19.4
7.3	15.5
8.0	11.3
8.8	17.1
11.1	19.7
13.0	17.9
18.5	18.7
21.3	24.6
24.2	15.2
27.0	13.6
62.1	26.1
66.5	16.4
101.4	26.2
119.2	16.5
130.7	18.9
135.4	31.3

In the outer suburbs, the relationship between *population* and *area* is non-linear.

A **log** transformation can be applied to the variable *area* to linearise the scatterplot.

- a. Apply the **log** transformation to the data and determine the equation of the least squares regression line that allows the population of an outer suburb to be predicted from the logarithm of its area.

Write the slope and intercept of this least squares regression line in the boxes provided below.

Round your answers to two significant figures.

2 marks

$$\text{population} = \boxed{} + \boxed{} \log(\text{area})$$

- b. Use the equation of the least squares regression line in **part a.** to predict the population of an outer suburb with an area of 90 km².

Round your answer to the nearest one thousand people.

1 mark

Question 5 (4 marks)

There is a negative association between the variables *population density*, in people per square kilometre, and *area*, in square kilometres, of 38 inner suburbs of the same city.

For this association, $r^2 = 0.141$

- a. Write down the value of the correlation coefficient for this association between the variables *population density* and *area*.

Round your answer to three decimal places.

1 mark

- b. The mean and standard deviation of the variables *population density* and *area* for these 38 inner suburbs are shown in the table below.

	<i>Population density</i> (people per km ²)	<i>Area</i> (km ²)
Mean	4370	3.4
Standard deviation	1560	1.6

One of these suburbs has a population density of 3082 people per square kilometre.

- i. Determine the standard z-score of this suburb’s population density.

Round your answer to one decimal place.

1 mark

- ii. Interpret the z-score of this suburb’s population density with reference to the mean population density.

1 mark

- iii. Assume the areas of these inner suburbs are approximately normally distributed.

How many of these 38 suburbs are **expected** to have an area that is two standard deviations or more above the mean?

Round your answer to the nearest whole number.

1 mark

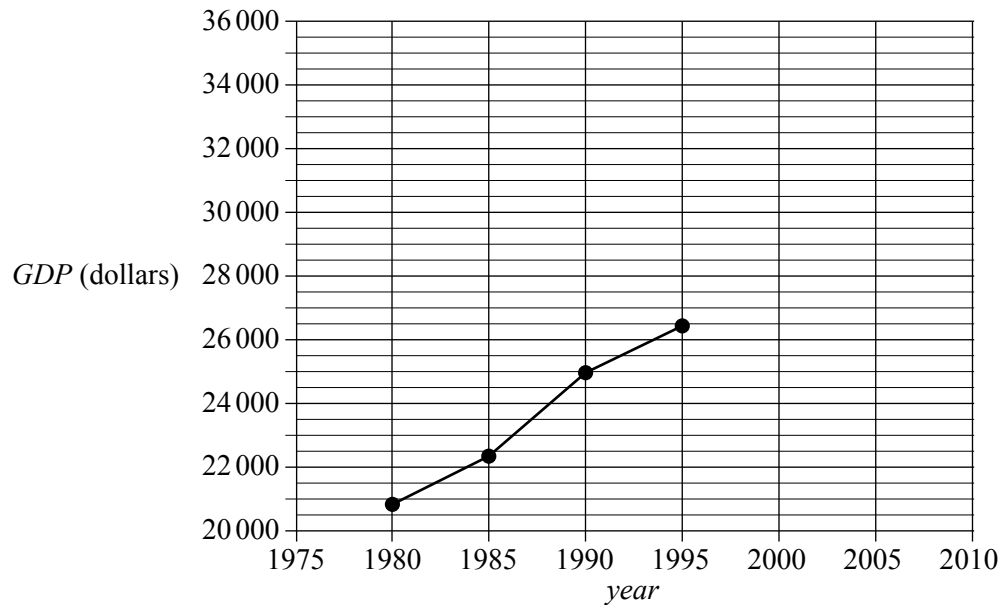
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Question 6 (5 marks)

Table 1 shows the Australian gross domestic product (*GDP*) per person, in dollars, at five yearly intervals (*year*) for the period 1980 to 2005.

Table 1

<i>Year</i>	1980	1985	1990	1995	2000	2005
<i>GDP</i>	20 900	22 300	25 000	26 400	30 900	33 800



- a. Complete the **time series plot above** by plotting the *GDP* for the years 2000 and 2005. 1 mark
- (Answer on the time series plot above.)*
- b. Briefly describe the general trend in the data. 1 mark

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- c. In Table 2, the variable year has been rescaled using 1980 = 0, 1985 = 5, and so on. The new variable is *time*.

Table 2

<i>Year</i>	1980	1985	1990	1995	2000	2005
<i>Time</i>	0	5	10	15	20	25
<i>GDP</i>	20 900	22 300	25 000	26 400	30 900	33 800

- i. Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the explanatory variable. 2 marks

- ii. The least squares regression line in **part c.i.** above has been used to predict the *GDP* in 2010. Explain why this prediction is unreliable. 1 mark

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Recursion and financial modelling

Question 7 (4 marks)

Hugo is a professional bike rider.

The value of his bike will be depreciated over time using the flat rate method of depreciation.

The value of Hugo's bike, in dollars, after n years, V_n , can be modelled using the recurrence relation below.

$$V_0 = 8400, \quad V_{n+1} = V_n - 1200$$

- a. Using the recurrence relation, write down calculations to show that the value of Hugo's bike after two years is \$6000.

1 mark

Hugo will sell his bike when its value reduces to \$3600.

- b. After how many years will Hugo sell his bike?

1 mark

The unit cost method can also be used to depreciate the value of Hugo's bike.

A rule for the value of the bike, in dollars, after travelling n kilometres is

$$V_n = 8400 - 0.25n$$

- c. What is the depreciation of the bike per kilometre?

1 mark

After two years, the value of the bike when depreciated by the unit cost method will be the same as the value of the bike when depreciated by the flat rate method.

- d. How many kilometres has the bike travelled after two years?

1 mark

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Question 8 (5 marks)

Hugo won \$5000 in a road race. He deposited this money into a savings account.

The value of Hugo’s savings after n months, S_n , can be modelled by the recurrence relation below.

$$S_0 = 5000, \quad S_{n+1} = 1.004 S_n$$

- a.** What is the annual interest rate (compounding monthly) for Hugo’s savings account? 1 mark

- b.** What would be the value of Hugo’s savings after 12 months? 1 mark

Using a different investment strategy, Hugo could deposit \$3000 into an account earning compound interest at the rate of 4.2% per annum, compounding monthly, and make additional payments of \$200 after every month.

Let T_n be the value of Hugo’s investment after n months using this strategy.

The monthly interest rate for this account is 0.35%.

- c. i.** Write down a recurrence relation, in terms of T_{n+1} and T_n , that models the value of Hugo’s investment using this strategy. 1 mark

- ii.** What is the total interest Hugo would have earned after six months? 2 marks

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Question 9 (3 marks)

Hugo needs to buy a new bike.

He borrowed \$7500 to pay for the bike and will be charged interest at the rate of 5.76% per annum, compounding monthly.

Hugo will fully repay this loan with repayments of \$430 each month.

- a. How many repayments are required to fully repay this loan?

Round your answer to the nearest whole number.

1 mark

After the fifth repayment, Hugo increased his monthly repayment so that the loan was fully repaid with a further seven repayments (that is, 12 repayments in total).

- b. i. What is the minimum value of Hugo's new monthly repayment?

1 mark

- ii. What is the value of the final repayment required to ensure the loan is fully repaid after 12 repayments?

1 mark

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**END OF SECTION A
TURN OVER**

SECTION B – Modules

Instructions for Section B

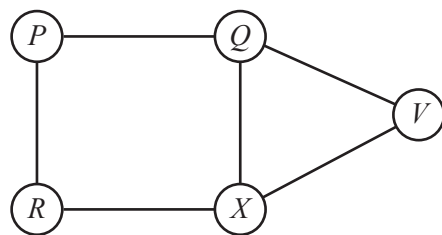
Select **two** modules and answer **all** questions within the selected modules. Write using blue or black pen.
 You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.
 Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1 – Matrices**Question 1** (2 marks)

Five trout-breeding ponds, P , Q , R , X and V , are connected by pipes, as shown in the diagram below.



The matrix W is used to represent the information in this diagram.

$$W = \begin{matrix} & \begin{matrix} P & Q & R & X & V \end{matrix} \\ \begin{matrix} P \\ Q \\ R \\ X \\ V \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

In matrix W :

- the 1 in row 2, column 1, for example, indicates that pond P is directly connected by a pipe to pond Q
- the 0 in row 5, column 1, for example, indicates that pond P is not directly connected by a pipe to pond V .

- a. In terms of the breeding ponds described, what does the sum of the elements in row 3 of matrix W represent?

1 mark

The matrix W^2 is shown below.

$$W^2 = \begin{matrix} & \begin{matrix} P & Q & R & X & V \end{matrix} \\ \begin{matrix} P \\ Q \\ R \\ X \\ V \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 2 & 1 \\ 0 & 3 & 2 & 1 & 1 \\ 0 & 2 & 2 & 0 & 1 \\ 2 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix} \end{matrix}$$

- b. Matrix W^2 has a 2 in row 2 (Q), column 3 (R).

Explain what this number tells us about the pipe connections between Q and R .

1 mark

Question 2 (10 marks)

10 000 trout eggs, 1000 baby trout and 800 adult trout are placed in a pond to establish a trout population.

In establishing this population:

- eggs (E) may die (D) or they may live and eventually become baby trout (B)
- baby trout (B) may die (D) or they may live and eventually become adult trout (A)
- adult trout (A) may die (D) or they may live for a period of time but will eventually die.

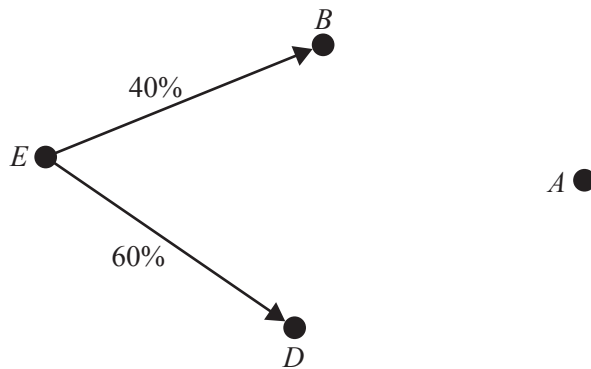
From year to year, this situation can be represented by the transition matrix T , where

$$T = \begin{matrix} & \begin{matrix} \textit{this year} \\ E & B & A & D \end{matrix} \\ \begin{matrix} E \\ B \\ A \\ D \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \end{matrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix} \textit{ next year}$$

a. Use the information in the transition matrix T to

- i. determine the number of eggs in this population that die in the first year 1 mark

- ii. complete the transition diagram below, showing the relevant percentages. 2 marks



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The initial state matrix for this trout population, S_0 , can be written as

$$S_0 = \begin{bmatrix} 10000 & E \\ 1000 & B \\ 800 & A \\ 0 & D \end{bmatrix}$$

Let S_n represent the state matrix describing the trout population after n years.

b. Using the rule $S_{n+1} = T S_n$, determine

i. S_1 1 mark

ii. the number of adult trout predicted to be in the population after four years.
Round your answer to the nearest whole number of trout. 1 mark

c. The transition matrix T predicts that, in the long term, all of the eggs, baby trout and adult trout will die.

i. How many years will it take for all of the adult trout to die (that is, when the number of adult trout in the population is first predicted to be less than one)? 1 mark

ii. What is the largest number of adult trout that is predicted to be in the pond in any one year? 1 mark

d. Determine the number of eggs, baby trout and adult trout that, if added to or removed from the pond at the end of each year, will ensure that the number of eggs, baby trout and adult trout in the population remains constant from year to year. 2 marks

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The rule $S_{n+1} = T S_n$ that was used to describe the development of the trout in this pond does not take into account new eggs added to the population when the adult trout begin to breed.

To take breeding into account, assume that every year 50% of the adult trout each lay 500 eggs.

The matrix describing the population after n years, S_n , is now given by the new rule

$$S_{n+1} = T S_n + 500 M S_n$$

where

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix}, M = \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } S_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

- e. Use this new rule to determine S_2 .

1 mark

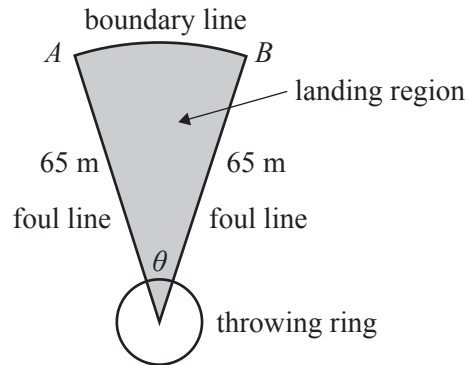
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Module 3 – Geometry and measurement

Question 1 (3 marks)

One of the field events at athletics competitions is the discus.

The field markings for the discus event consist of a circular throwing ring, foul lines and the boundary line of the field, as shown in the diagram below. The shaded area on the diagram is the landing region for a discus throw.



The foul lines meet the boundary line at points A and B , 65 m from the centre of the throwing ring.

The angle θ is 34.92° .

- a. What is the length of the boundary line from point A to point B ?

Write your answer in metres, rounded to two decimal places.

1 mark

- b. Calculate the area of the landing region.

Round your answer to the nearest square metre.

2 marks

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Question 2 (5 marks)

Daniel lives in Mildura (34° S , 142° E). He will fly to Sydney (34° S , 151° E) and then fly on to Rome (42° N , 12° E) to compete in the discus event at an international athletics competition.

In this question, assume that the radius of Earth is 6400 km.

- a. Find the shortest great circle distance to the South Pole from Mildura (34° S , 142° E).

Round your answer to the nearest kilometre.

1 mark

- b. The flight from Mildura (34° S , 142° E) to Sydney (34° S , 151° E) travels along a small circle.

- i. Find the radius of this small circle.

Round your answer to two decimal places.

1 mark

- ii. Find the distance the plane travels between Mildura (34° S , 142° E) and Sydney (34° S , 151° E).

Round your answer to the nearest kilometre.

1 mark

DO NOT WRITE IN THIS AREA

- c. How long after the sun rises in Sydney (34° S, 151° E) will the sun rise in Rome (42° N, 12° E)?

Round your answer to the nearest minute.

1 mark

- d. Daniel's flight to Rome leaves Sydney airport on Sunday, 6 March at 10.20 am, local time. The flight arrives in Rome on Monday, 7 March at 2.30 am. Assume the time difference between Sydney and Rome is 10 hours.

How long does the flight take to travel from Sydney to Rome?

Round your answer to the nearest minute.

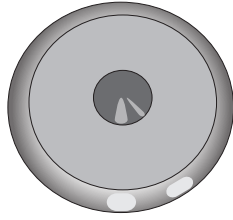
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Question 3 (2 marks)

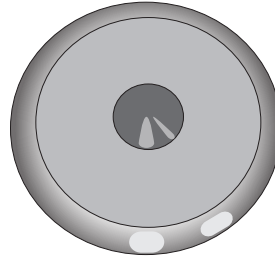
Daniel will compete in the intermediate division of the discus competition. Competitors in the intermediate division use a smaller discus than the one used in the senior division, but of a similar shape. The total surface area of each discus is given below.

Intermediate discus



total surface area 500 cm^2

Senior discus



total surface area 720 cm^2

By what value can the volume of the intermediate discus be multiplied to give the volume of the senior discus?

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Question 4 (2 marks)

Daniel has qualified for the finals of the discus competition.

On his first throw, Daniel threw the discus to point A , a distance of 53.32 m on a bearing of 057° .

On his second throw from the same point, Daniel threw the discus a distance of 57.51 m.

The second throw landed at point B , on a bearing of 125° , measured from point A .

Determine the distance, in metres, between points A and B .

Round your answer to one decimal place.

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